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AUTOMATIC NETWORK DETECTION

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INTRODUCTION

The performance of an F-detector on a 31-element TFO short-period array has been studied by Blandford (1970). He found that the detector operated in good agreement with theory; therefore it seems reasonable to study the expected performance of an F-detector operating on a network. This would be of application to at least two distinct types of systems: (1) an array such as NORSAR where signal correlation between subarrays may be poor; in this case a detector might be operated independently for each subarray, and the results combined in some way; (2) a world-wide network of completely independent arrays with different noise and signal levels.

A simple F-detector is said to detect if $F(N_1,N_2,\lambda) \geq F_0$, where $F(N_1,N_2,\lambda)$ is the computed test statistic, assumed to belong to a population distributed as a non-central F with N_1 and N_2 degrees of freedom and noncentrality parameter λ, and F_0 is the threshold determined from the desired false alarm rate. Thus the probability of detection by this detector is the probability that $F \geq F_0$ in the presence of a signal. In order to make meaningful comparisons between different systems, the false alarm rate should be held constant, and this is done in all the measurements made in this report. Blandford (1970) suggests that $N_1 = 2BT = 3$ is suitable, where integrated bandwidth = B = 0.5 Hz, and the signal window = T = 3.0 seconds. Theory shows that $N_2 = 2BT(N-1)$, where N is the number of elements in the array. (Some notational confusion is possible with the denominator of the expression for signal-to-noise ratio (S/N); however, this is usually clear from the context.)

A power detector may be defined as the ratio of the beam power in a short signal window to that in an arbitrarily long noise window. If the noise is stationary this is equivalent, for small signals, to an F detector with N_2 arbitrarily large, approaching a χ^2 distribution. Given the hypothesis of stationary noise, this would be superior in pure detection capability to the F-detector, where the noise statistics are accumulated only in the signal window, and where N_2 is therefore limited. However, Blandford (1970) showed that the F detector has features which make it more reliable in practice, and in this paper we show that by use of long time windows the quantitative difference can be made negligible for all except the smallest arrays of 2-3 elements.

Signal-to-noise values used throughout this report are calculated as the square root of power ratios. Blandford showed that to convert these values to those given by the ratio of peak-to-peak signal divided by peak-to-peak noise in the previous 60 seconds, the square root of the power ratio should be multiplied by 0.5. Signal-to-noise values are defined at both the instrument and beam level by the symbols $(S/N)_{seis}$ and $(S/N)_{beam}$ respectively. The relation $(S/N)_{beam} = N^{1/2}(S/N)_{seis}$ is true since the noise is uncorrelated, by hypothesis for the F-detector.

In the following sections, we first discuss the question of the optimum signal window length over which to calculate the F statistic. Blandford used 3 seconds, but this is shown not to be optimum for small arrays.

We then develop the theory for a "composite F-detector" in which a vote is taken among independently detecting arrays in order to settle on a final detection.

This is followed by a section in which the "multi-array F-detector" is developed. By this technique the actual traces from each independent array are combined into a single statistic which for uncorrelated signals is, in theory, superior to any other detector, including the composite detector.

The two methods are then extended to the case of unequal expected signal-to-noise ratios at the different subarrays. This is, of course, particularly applicable to a world-wide network of stations some of which would be closer to an epicenter than others. However, many workers have also shown that for small seismic regions of the earth there are significant repeatable amplitude anomalies recorded across LASA, so the two methods would also be suitable detectors for such a case.

OPTIMUM SIGNAL WINDOW FOR DETECTION

Blandford (1970) used a 3-second signal window in his study of the automatic detector at TFO. This window was suggested by the studies of earlier workers and by intuition, which suggests that the window should be only as long as the dominant portion of the signal. In his Figure 8, Blandford showed that for small arrays, $N < 7$, there were severe departures from proportionality to $N^{1/2}$ for the signal-to-noise ratio which would be detected 90 per cent of the time.

It seemed plausible that this departure occurred because the noise estimate, calculated in the signal window by averaging residual power over all N channels, was not statistically reliable for small N. The stability would be increased by a longer time window, but the signal-to-noise ratio would decrease by $(3/T)^{1/2}$. It turns out, as we shall see below, that the two effects almost perfectly cancel for large arrays, and that detection is substantially improved for small arrays.

As T increases, $\lambda = 2BT(S/N)^2_{beam}$ remains constant, since $(S/N)^2 \sim 1/T$. N_1 and N_2 increase in proportion to T/3. Calculation for successive values of T yields the curves in Figure 1. In the neighborhood of the optimum window length the threshold signal is insensitive to the precise value of the window length. However, for 3 elements there is almost 0.5 m_b improvement in the required signal between the 3-second and 30-second windows.

The optimum window length as a function of the number of elements is shown in Figure 2. The actual optimum length is presumably somewhat longer since in reality the signal is not exactly zero outside of a 3-second window.

In Figure 3 the upper curve gives the element signal/noise value required for 90 per cent probability of detection at 0.1 false-alarm per day for a 3 second time window as a function of the number of elements in the array. The lower curve gives the signal/noise value for the optimum window. The curves are shifted up by a factor of $\sqrt{2}$ over those given by Blandford (1970) due to an error in calculation in that paper.

We note that both curves are asymptotic to $N^{1/2}$, the theoretical performance curve for a power detector. This can be simply explained by noting that the signal/noise ratio required on the beam for a given probability of detection decreases as a function of N, but approaches a limit for large N, for which further increases of N_2 have little effect. Thus for large N, the decrease of seismometer (S/N) required results primarily from the $N^{1/2}$ improvement of (S/N) on the beam.

AT LEAST K SUBARRAY DETECTION

When the signal is not identical across a large array, as may be the case at NORSAR, the array can be broken into a number of smaller subarrays and the F detector operated separately on each subarray. If one then asks for at least K subarrays to detect before one declares a detection (K is a number yet to be specified) then the detection capability is not much worse than one would obtain in the ideal case of an identical signal on the entire array acting as one F detector. It turns out that there is an optimum choice of K which maximizes the signal detection probability with fixed false-alarm rate. This value is usually slightly more than half the number of subarrays, and it depends on the numbers of degrees of freedom (i.e., on the time and frequency windows and the number of elements per subarray) and on the number of subarrays, also to a slight extent on the chosen false-alarm rate (F threshold) and signal level. The maximum is, however, fairly broad and an error of 1 or 2 in the choice of K would not seriously degrade the performance of a large system.

We assume in our analysis that the signals are perfectly correlated across each individual subarray (and the noise perfectly uncorrelated). In addition, we will begin by assuming the signals on different subarrays to be of the same size (same signal/noise ratio), although later we will consider the effects of different size signals. Representing the probability of a detection on any subarray by p (the same for all subarrays), the probability of at least K detections out of M is given by the summed binomial distribution

$$P(\geq K) = \sum_{i=K}^{M} \binom{M}{i} p^i (1-p)^{M-i} \tag{1}$$

This formula is used to evaluate either the false-alarm rate or the signal detection probability, and p is accordingly computed from a central F or a non-central F, respectively. Subroutines to do these calculations are given in the Appendix. Another assumption we have made in our calculations here is that the frequency window is 1/2 hertz, so that $N_1 = T$, i.e. the number of degrees of freedom of the numerator equals the time window in seconds, an approximation to a system already in operation (Blandford, 1970). In any case, the non-centrality parameter is $\lambda = N N_1 (S/N)^2$, where (S/N) is the signal/noise ratio on each element and N is the number of elements/subarray. In expressing our false-alarm rate in terms of false-alarms/day we have assumed 86400/T samples per day, although if an overlapping or sliding window is used (the usual case), there will presumably be an additional as-yet-undetermined factor. In view of the steepness of the operating curves obtained, this is probably not a crucial point.

We have evaluated this detector for certain configurations that might be used at NORSAR: 6 elements/subarray (N_1 = 3 and N_2 = 15) and 7, 13, and 22 subarrays.

Figures 6 through 8 show the effect of choice of K on signal detection vs false-alarm rate for a fixed signal strength. Figures 9 through 11 are operating curves for the optimum K detectors for the same configurations. Table II summarizes these results in terms of signal/noise ratios on subarray beams for 90 per cent detection probability with one false-alarm/day and compares them with the performance of a single subarray and with the ideal performance of a single F detector operating on the whole array ("total beam"), given in Table I. Also included in Tables I and II are similar data for groups of 12-element subarrays ($N_1 = 3$ and $N_2 = 33$). Figures 4 and 5 give operating curves for single subarrays. (Note that element signal/noise ratios are derived from beam signal/noise ratios by dividing by the square root of the number of elements in the subarray.) Table II shows that the optimum K detector is significantly superior to a single subarray, and in the case where the total array is so large that significant loss of signal coherence occurs, it may well perform better than a single F detector operating on the total beam.

In view of the result obtained above, that using a time window somewhat longer than the three seconds average signal length gave better results with a single detector, we tried using a longer time window on the composite detector. The results, shown in Table III, are negative, even in the case of only three elements/subarray. Apparently the composite detector is so much more sensitive to signal/noise ratio than to degrees of freedom that the improvement in noise estimate obtained with a longer window is insufficient to offset the degradation in effective signal/noise ratio. We have assumed a signal exactly three seconds long, so the effective S/N ratio will be degraded by a factor $\sqrt{3/T}$. Of course in practice there will usually be some contribution to the signal after three seconds, but it is probably still true that the best time window for a network of F detectors corresponds to the effective signal duration, even though the best time window for an individual F detector may be somewhat longer. Thus in future systems there may have to be a compromise between the role of a station as a station and its role as part of a network, but the difference, in any case, is not very great.

For network applications there is another consideration that has a possible bearing on the choice of time window and on the way decisions are reached. A calculation based on (1) shows that for a 22-station system, a network false-alarm rate of 1 per day implies an average false-alarm rate of ~4600/day for each individual station. Furthermore, decreasing the network false-alarm rate to 10^{-6}/day decreases the station false-alarm rate only slightly, to ~1300/day, due to the K'th power in (1). Thus the individual station detections, by themselves, would be relatively useless for decision making, at the thresholds used for network detection. Also, experimentally determining the proper delays to correlate individual "detections" would seem impractical because of the large number of combinations. The best approach would seem to be a fully automatic detector in which "all possible beams" are scanned to cover a given area. In practice this would merely involve scanning a table of delay times and counting votes over sections of data corresponding to the greatest and least delays for each station for that region. It does not seem at all unlikely that a region the size of Russia could be monitored in real time with existing computers. With an efficient procedure much larger areas could probably be scanned. The size of each beam and hence the number necessary to cover a given area, would be influenced by the time window used: a longer time window would reduce the number required. Notice that automatic network detectors necessarily give approximate locations simultaneously with the detections. Most of these remarks also apply to the multi-array F detector described below.

We should also point out that false-alarm rates quoted in this report are <u>per beam</u> and must be multiplied by the number of beams to give the total network false-alarm rate. For a system operating a thousand beams, the rates quoted here are certainly too high. False-alarm rates down to 10^{-2}/day can be read directly from the operating curves, and linear extrapolation is satisfactory. For a 22 station system, threshold magnitude increases ~.6dB/decade decrease in false-alarm rate. This represents another way in which time window length can influence detection thresholds in real systems.

A variation of the composite detector was also investigated: on receipt of at least k_1 subarray detections at threshold F_1, the threshold on the remaining subarrays is lowered to F_2 and a total of at least k_2 detections is then required in order to declare a detection by the system. Writing $P(\binom{N}{k}|p)$ to represent the probability of exactly k events out of N when individual events have probability p, a shorthand for the binominal distribution, the probability of at least k_2 subarrays detecting by this scheme is

$$P(\geq k_2) = P(\binom{N}{\geq k_2}|p_1) + \sum_{i=k_1}^{k_2-1} P(\binom{N}{i}|p_1)\, P(\binom{N-i}{\geq(k_2-i)}|p_2-p_1) \qquad (2)$$

where p_1 is the subarray detection probability at threshold F_1, p_2 is the probability at threshold F_2, and p_2-p_1 is the fraction lying between F_2 and F_1. (We use the symbol " $\geq$ " to mean "at least".) Note that $P(\geq k_2) \leq P(\binom{N}{\geq k_1}|p_1)$. This scheme was always worse than the previous, and much simpler, detector. It gave comparable results only in the limits $k_1 \rightarrow k_2$ and/or $F_1 \rightarrow F_2$, which reduce to the previous case. (False-alarm rate was always held constant; the ratio F_2/F_1 was specified and F_1 determined by an inversion.) The reason for the poorer performance is probably that the effective number of subarrays is reduced by using a lower K value and by exaggerating the weak subarrays: the statistical "inertia" of the system is reduced by the manipulations. Schemes of this type appear to be extremely unpromising and the idea has not been pursued further. It might still be worthwhile to review non-detecting stations for arrival time information in order to obtain a better least-squares location, although it may very well be that using the arrival times on such stations would not actually produce a better location. This is particularly apt to be true for large K, where one is practically guaranteed a good location for all detections anyway.

A MULTI-ARRAY F DETECTOR

There is another approach possible when signals are not the same on all subarrays, which is a special case of a technique of Shumway (1970). We consider the model representing the output from the i'th sensor on the j'th subarray as

$$y_{ji}(t) = a_j S_j(t) + n_{ji}(t)$$

where $j = 1, \ldots, M$; $i = 1, \ldots, N$; and $t = 0, 1, \ldots, L-1$. We assume the noise $n_{ji}(t)$ to be normal, stationary, and uncorrelated between sensors, and the signals S_j to be the same on all sensors of a given subarray. We further assume the S_j all to have the same rms value over time and explicitly allow for different size signals by inclusion of the factors a_j, which we assume are known <u>a priori.</u> Then, following Shumway (1970), the maximum-likelihood estimate for the signal on the j'th subarray is just

$$\hat{S}_j(t) = a_j^{-1}\, \bar{y}_{j\cdot}(t)$$

where the dot signifies the subscript over which the mean is taken.

In order to test the hypothesis that there is no signal present on any subarray, i.e., that all $S_j(t) = 0$, we could transform to the frequency domain and form the ratio of beam power to residual power as our test statistic. But if the data are band-limited, to bandwidth B, over which the noise spectrum is constant, we can instead use an approximation to the F statistic given by

$$F_{2BTM,2BTM(N-1)} \approx \frac{(N-1)N \sum_j \sum_t \bar{y}_{j\cdot}^2(t)}{\sum_j \sum_t [\sum_i y_{ji}^2(t) - N\,\bar{y}_{j\cdot}^2(t)]} \tag{3}$$

where B is the filter bandwidth in hertz, T the time window in seconds, M the number of subarrays, and N the number of elements/subarray. In the presence of signal, the non-centrality parameter is

$$\lambda(\omega_o) = \frac{2BTN \sum_{j=1}^{M} a_j^2 |\bar{S}(\omega_o)|^2}{N^2(\omega_o)} \tag{4}$$

where

$$|\bar{S}(\omega_o)|^2 = \frac{\sum_{j,t} a_j^2 S_j^2(t)}{BT \sum_j a_j^2}$$

is the approximate mean signal power within the frequency band, and $N^2(\omega_o)$ is the noise power. We include the filter center frequency ω_o in (4) in order to emphasize the dependence on filter characteristics; in all of the work in this report, we assume a filter similar to that used at TFO, as described in Blandford (1970).

We can abbreviate (4) to

$$\lambda = N N_1 \left(\sum_{j=1}^{M} a_j^2/M \right) (S/N)^2 \qquad (5)$$

where $N_1 \equiv 2BTM$ is the number of degrees of freedom of the numerator and (S/N) is the normalized signal/noise ratio, defined, when $a_j = 1$, as in Blandford (1970). When all $a_j = 1$, the expression for λ has the same form as previously, although the statistic is, in general, different. When M = 1, the detector reduces to the single-array F detector.

In order to evaluate the performance of this detector on an array such as NORSAR, where the signals are generally of the same size on all subarrays, we set all $a_j = 1$. Table IV gives the results for some of the same configurations for which the previous detector was evaluated. The multi-array F detector does somewhat better but is still worse than the ideal total array beam. Figures 12 through 14 give operating curves for this detector. Results obtained for a six-second time window were worse, indicating that there, too, the optimum length is about three seconds for a three second signal. Table V gives operating thresholds (F values) for all the systems described.

UNEQUAL SIGNAL AMPLITUDES

The multi-array F detector, as presented here, could also be used on a worldwide network of stations. We have so far assumed the noise, as well as the signal, to have the same rms amplitude on all arrays. When this is not the case, we first normalize the traces by a long-term average of the noise estimates. The average should be long enough not to affect the statistics and short enough to follow daily variations in the noise level. In practice, a running average of the estimated residual noise power from the preceding hour or so would probably be satisfactory. The theory assumes the noise to be identically distributed on all channels, but amplitude variations can be treated as an amplifier gain error. Notice that in forming the network test statistic (3), the sums over i and t can be done at each station prior to transmission to the central station, so that the only quantities that need to be transmitted are the contributions to the j sums in the numerator and denominator, i.e., just two numbers. (The contributions to the denominator will average close to 1, due to the noise normalization, but the fluctuations are important.) The central station then has to sum contributions from windows with the appropriate time delays, divide, and compare with the threshold.

For purposes of analyzing the performance of such a network, the amplitude factors a_j would be given by the distance-amplitude corrections for each station for a given epicenter location, divided by the rms noise.

Relation (5) says that λ, and hence the detection capability, depends on the root-mean-square of the amplitude factors. Thus curves such as given in Figures 12 through 14 can be used here also by multiplying the signal/noise ratio indicated in the Figures by $a_j(\sum a_j^2/M)^{-1/2}$ to obtain the signal/noise ratio required on each station. Note that the false-alarm rate does not depend on the a_j, so Table V can also be used for this case.

The composite ($\geq K$) detector can also be used for a worldwide network, and since there are no special requirements on the noise at different stations, no normalization need be done. In addition, the summation at the central station should be somewhat faster than in the case of the multi-array F detector, so that more beams could be formed in real time. Also, the only information that needs to be transmitted to the central station is a 1 or a 0. The difficulty with this detector lies in the analysis of its performance, which is troublesome but not insurmountable. The first thing to notice is that, with unequal signal strengths, the probabilities of detection on each individual station, p_j, will no longer be equal as required by (1). Thus to compute $P(\geq K)$, a more complicated procedure must be used (Wirth, 1971). The false-alarm rate can be computed as before if all the stations have the same number of elements, although this is not a requirement for the detector to work. (One could require that each station have the same false-alarm rate, and then set different thresholds F_j for stations with different numbers of elements, or degrees of freedom. Numerically this would be easy to do and might also possess some statistical advantage over the scheme of using the same F threshold on each station, although this has not been investigated.) The principal difficulty lies in the fact that the optimum choice of K depends on the relative signal strengths at the different stations, as will be shown below. Thus for optimum performance, different choices of K will in general be required for monitoring different regions of the earth. There would be no particular problem in implementing this, since K is just the minimum number of individual "detections" required in order for the central collecting station to proclaim a network detection, and different numbers could easily be programmed for monitoring different regions of the earth. A complication arises because the false-alarm rate also depends on K, so one would either have to allow the false-alarm rate to vary for different epicenter regions or else specify different station thresholds also. Again, this would not be difficult to implement. Thus the composite ($\geq$K) detector is somewhat more flexible, slightly less sensitive, and requires a lot more individual tailoring than the multi-array F detector for network applications.

To investigate the behavior of the optimum K for differing signal levels, we consider a simple case in which there are only two different signal levels. We take M_1 stations with beam signal/noise ratio $(S/N)_1$ and M_2 stations with ratio $(S/N)_2$, such that $M = M_1 + M_2$ is constant. Overall false-alarm rate is held constant. Figure 15 is a contour plot of the optimum K values, holding $(S/N)_2$ constant and allowing $(S/N)_1$ to vary geometrically. (The surface should be considered to be composed of terraces. See Wirth, 1971, for a description of the contour algorithm used.) The figure shows that the general effect of having a few stations with large signal/noise ratio is to decrease the value of K. The figure also makes it obvious that guessing the optimum K would be a little difficult, even in this simplified case. Given an actual network, however, it would not be difficult to calculate the optimum strategies. Figure 16 gives contours of the corresponding probability of detection for the optimum choices of K. The increase towards the upper right corner is not really surprising.

An interesting comparison can be made with the multi-array F detector by holding the rms of all the signals constant, instead of $(S/N)_2$. Since the F detector is sensitive only to the root-mean-square of the signal/noise ratios on all the stations, the probability of detection should be constant. Figures 17 and 18 are the corresponding plots for this constraint. The rms signal/noise ratio is 1.224, for which the multi-array F detector has a 90 per cent probability of detection. The behavior of K is very similar to the preceding case, indicating that it is sensitive primarily to the ratios of signal levels. The behavior of the probability of detection for the optimum choice of K (Figure 18) is, however, not entirely expected. It should be remembered that the multi-array F detector should have 90 per cent probability of detection over the whole plot. This points up another difference between the two detectors: while for equal signal levels the composite ($\geq K$) detector is nearly as good as the multi-array F detector, for disparate signals it does increasingly worse. In particular, Figure 18 shows that the presence of a few stations with large signal/noise ratio is especially damaging in comparison. This should be kept in mind when making a choice between the two detectors for any particular system, however the difference may not be as great as it seems. The 66 per cent contour represents a 1 dB difference in signal detection thresholds between the two detectors (Table IV) and the 30 per cent contour probably represents only an additional 1 dB, judging from the operating curves.

We have so far based our analysis on the assumption that all stations were to be retained, no matter how poor. This is not necessarily the way things are done in real life, and a little reflection shows that this is not always wise. For the composite detector it is easy to see why: very poor stations contribute only false alarms and thus represent a negative asset. The same thing is true of the multi-array F detector, but the reason is not quite so obvious. The work on optimum time windows for this detector shows that increasing the degrees of freedom while holding λ constant results in poorer performance unless N_1 and N_2 are very small. Equations (3) and (4) say that this is exactly what happens when one adds very weak stations to the network ($a^2_{M+1} << \sum a^2_j$).

The computations done above for the composite detector were repeated, throwing out the poor stations. Figures 19 and 20 are analogous to Figures 15 and 16 and give the contours of optimum K and the probabilities of detection for varying numbers of strong stations (M) and signal/noise ratio (S/N). Calculations were also done extending the graphs downwards, i.e., for the case of a few weak stations. These calculations show that it is generally better to discard stations whose signal/noise ratio is less than about .4 - .6 times the signal/noise ratio on the strong stations, if $M \geq 3$. (For example, comparing Figures 16 and 20 along the lines M_1 or M = 6 shows a crossover near $(S/N)_1/(S/N)_2 = 2.0$, above which it is better to drop the weak stations.) Considerations about the minimum desirable number of stations in the network for determining locations will also play a part here. If half the stations (11) have only 1/4 the signal/noise ratio on the other half (which is 1.5), then the probability of detection is ~60 per cent if the weak stations are dropped, compared with ~30 per cent if they are retained, which corresponds to a "threshold magnitude" change of about 1 dB. (In Figures 16 and 20, the greatest difference would be in the upper right corner, but since the probabilities are close to 1 here anyway, the difference is not as great as in the example cited.) Quantitative studies of the multi-array F detector for this case are being carried out in the context of determining the optimum filter, which is an analogous problem, as pointed out by Blandford. These will be reported in the future. Preliminary results indicate a rejection criterion similar to that for the composite detector.

CONCLUSIONS

1. A signal time window longer than 3 seconds improves the detection capability of F detectors on small arrays, resulting in performance closely approaching that of a power detector, without degrading the desirable features of the F detector.

2. These longer windows are a disadvantage, however, when the arrays are used in a detection network. Qualitatively this is because more than one station must trigger for a detection, so that a false alarm on only one due to statistical fluctuation is not serious. In practice both the short and long windows might be used, the long for detection at each station independently; and the short for the final decision in combining the station detections. After detection, a resurvey of the data could be performed for location purposes.

3. A full 22-subarray NORSAR can perform within 5.4 dB of its theoretical $N^{1/2}$ performance for perfect signal correlation by using independent F detectors on each 6-element subarray and declaring a detection with 12 (the optimum number) or more subarray detections. A multi-array F detector would lose only 4.3 dB. An iterative technique in which a first detection is made with a higher threshold on fewer than K subarrays, followed by detection using a lower threshold on more than K, is uniformly worse than the simple method.

4. If some stations have higher signal-to-noise values than others, the advantage of the multi-array detector over the composite detector becomes somewhat greater.

5. The multi-array detector requires slightly more data transmission, but the decision strategy for the composite detector is more difficult to calculate.

6. The different network detectors can be implemented for a large-aperture array, or a real world-wide network, by straightforward extrapolation of the techniques presented in this paper.

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TABLE I

Detection levels for total beams, assuming signals perfectly correlated. (Signal/noise ratios required for 90% detection at 1 false-alarm/day, with 3 second time window and $N_1 = 3$).

No. of Subarrays	No. of Elements	Beam S/N	Element S/N	Improvement Over Single Subarray
1	6	5.31	2.17	0 dB
7	42	3.583	.553	+11.9
13	78	3.505	.397	+14.7
22	132	3.47	.302	+17.1
1	12	4.13	1.193	0
4	48	3.57	.515	+ 7.3
6	72	3.52	.415	+ 9.2
11	132	3.47	.302	+11.9

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TABLE II

Detection levels for at least K subarray detection, K optimum. (Signal/noise ratios required for 90% detection at 1 false-alarm/day, with 3 second time window and $N_1 = 3$.)

N No. of Elements/ Subarray	M No. of Subarrays	NM Total No. of Elements	K	Beam S/N	Element S/N	Improvement Over single Subarray	Loss Compared w/Total Beam of NM Elements	Loss Compared w/Beam of 132 Elements
6	7	42	5	2.11	.862	+ 8.0 dB	-3.9 dB	-9.1 dB
6	13	78	8	1.673	.683	+10.0	-4.7	-7.1
6	22	132	12	1.388	.567	+11.6	-5.4	-5.4
12	4	48	3	2.43	.701	+ 4.6	-2.7	-7.3
12	6	72	4	2.09	.603	+ 5.9	-3.3	-6.0
12	11	132	7	1.683	.486	+ 7.8	-4.1	-4.1

TABLE III

Effect of larger time window on $\geq K$ subarray detection, K optimum.
(Signal/noise ratios required for 90% detection at 1 false-alarm/day, with N_1 = T and signal exactly 3 seconds long.)

N No. of Elements/ Subarray	M No. of Subarrays	K	T Time Window	Beam S/N	Element S/N	Required Element S/N	S/N Adjusting Factor	Loss Compared w/3 Second Window
6	7	5	3 sec.	2.11	.862	.862	1	0 dB
6	7	5	6	1.60	.653	.924	$\sqrt{2}$	- .6
6	7	5	12	1.227	.501	1.002	2	-1.3
6	13	8	3	1.673	.683	.683	1	0
6	13	8	6	1.292	.528	.746	$\sqrt{2}$	- .8
6	13	7	12	1.007	.411	.822	2	-1.6
6	22	12	3	1.388	.567	.567	1	0
6	22	12	6	1.088	.444	.629	$\sqrt{2}$	-1.0
6	22	12	12	.860	.351	.702	2	-1.9
3	22	14	3	1.55	.895	.895	1	0
3	22	13	12	.933	.539	1.08	2	-1.6
3	22	12	27	.710	.41	1.25	3	-2.8

TABLE IV

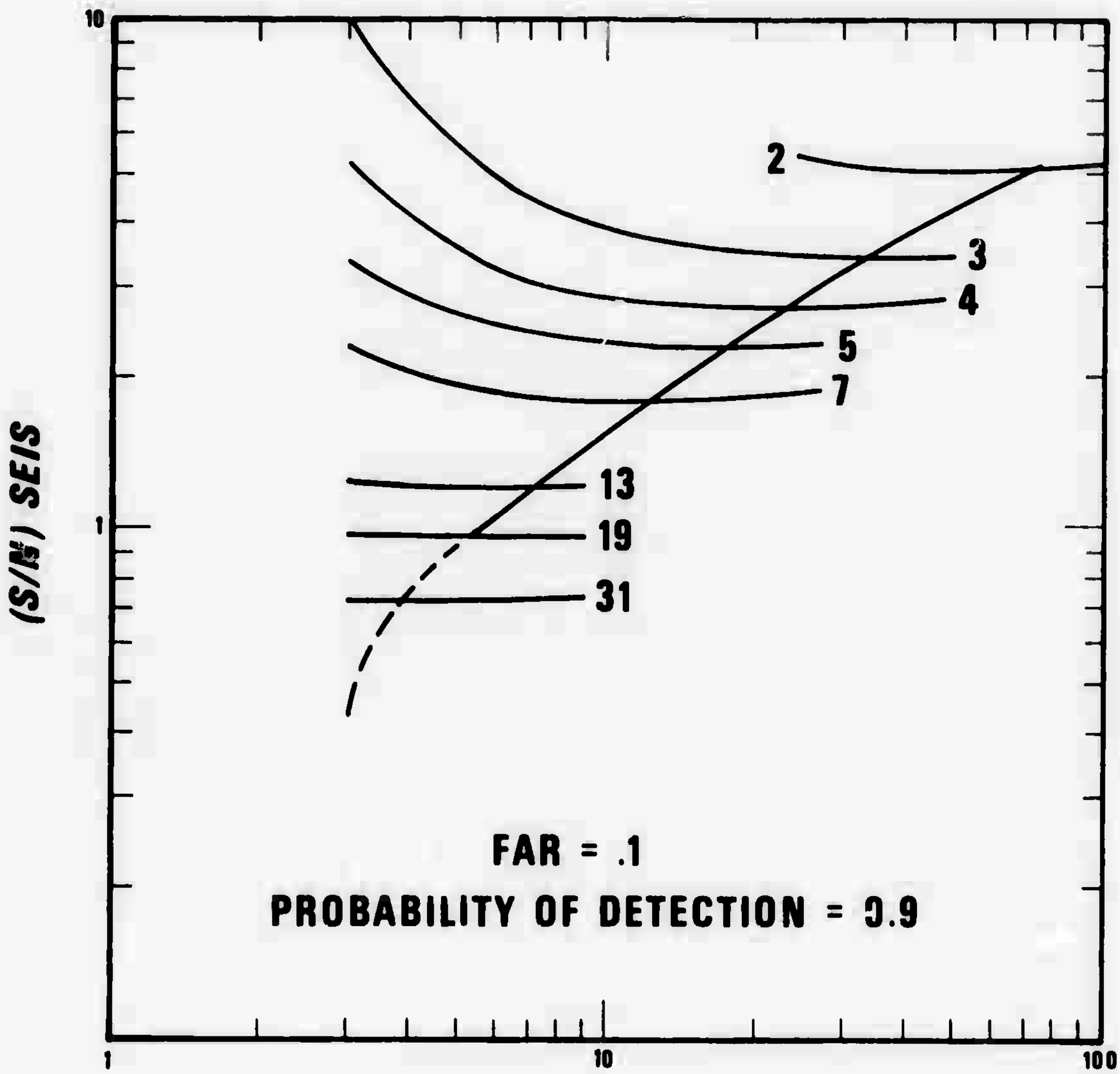
Detection levels for multi-array F detector, signals equal. (Signal/noise ratios required for 90% detection at 1 false-alarm/day, with 3 second time window.)

N No. of Elements Subarray	M No. of Subarrays	N_1	N_2	Beam S/N	Element S/N	Improvement Over Single Subarray	Loss Compared w/Total Beam of NM Elements	Improvement Over >K Subarray Detector
6	7	21	105	1.873	.765	+ 9.0 dB	-2.8 dB	+1.0 dB
6	13	39	195	1.473	.602	+11.1	-3.6	+1.1
6	22	66	330	1.224	.500	+12.7	-4.3	+1.1

TABLE V

Operating thresholds (F values) for systems described (3 second time window).

N_1	N_2	No. of Elements/ Subarray	No. of Subarrays	K	False-alarms/day 10.	1.	.1	.01
3	15	6	7	5	2.335	2.868	3.423	4.004
3	15	6	13	8	1.938	2.276	2.615	2.958
3	15	6	22	12	1.750	1.989	2.223	2.455
3	33	12	4	3	2.994	3.726	4.482	5.266
3	33	12	6	4	2.563	3.108	3.659	4.224
3	33	12	11	7	1.882	2.201	2.514	2.827
21	105	6	7	--	2.763	3.253	3.746	4.249
39	195	6	13	--	2.151	2.436	2.712	2.985
66	330	6	22	--	1.822	2.011	2.190	2.364



WINDOW IN SECONDS

Figure 1. Signal-to-noise values required at the seismometer level for a false alarm rate of 0.1 per day and probability of detection of 0.9 as a function of the signal window in seconds. Curves are shown for arrays ranging from 2 to 31 elements, and another curve has been drawn through the minima.

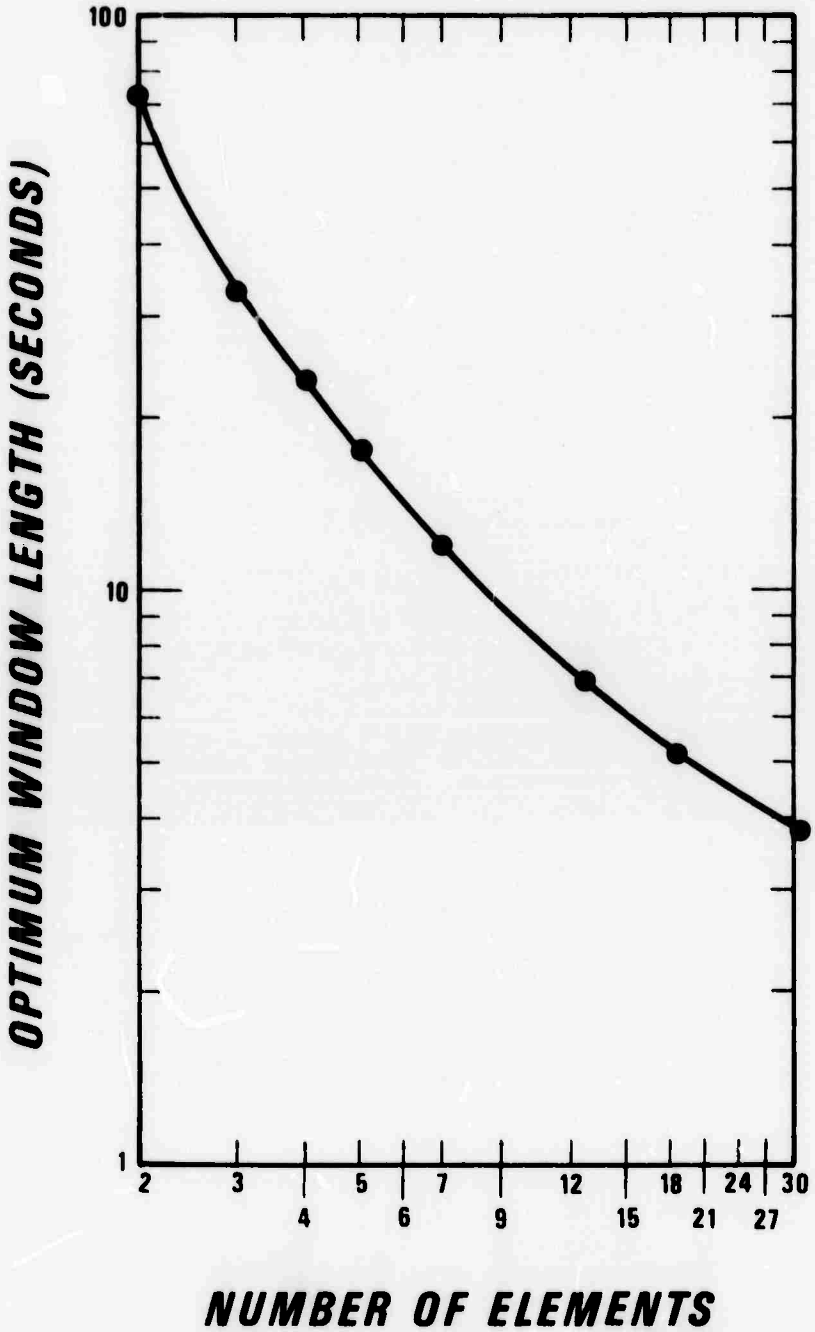


Figure 2. Optimum signal window for detection by a single array as a function of the number of elements in the array.

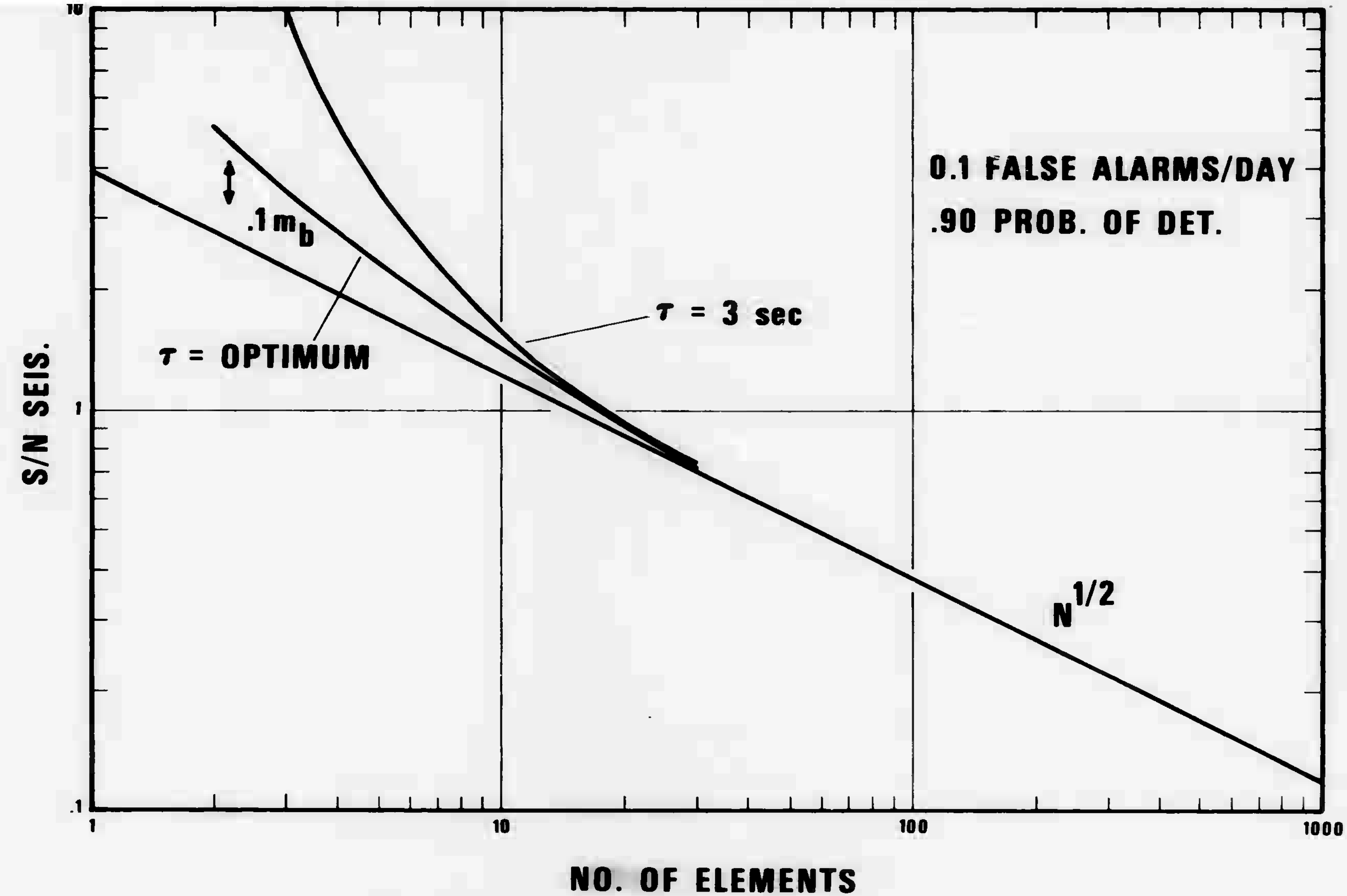


Figure 3. Signal-to-noise values required at the seismometer level for a false alarm rate of 0.1 per day and probability of detection of 0.9 as a function of the number of elements in the array. The upper curve is for a single window of 3 seconds, and the middle curve is for the optimum window shown in Figure 2. The lower curve shows signal to noise proportional to $N^{1/2}$.

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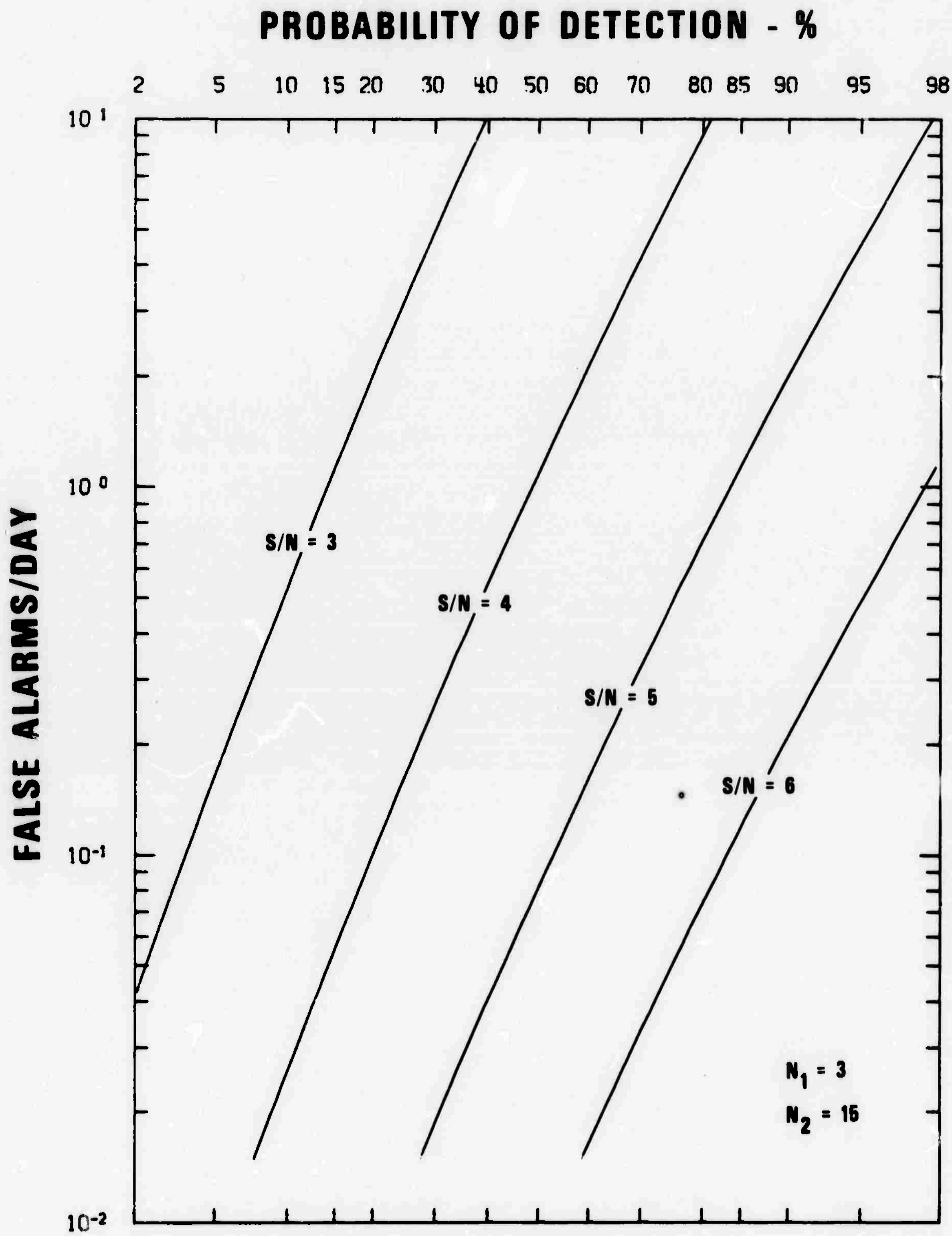

Figure 4. Probability of detection versus beam signal/noise, single subarray of 6 elements.

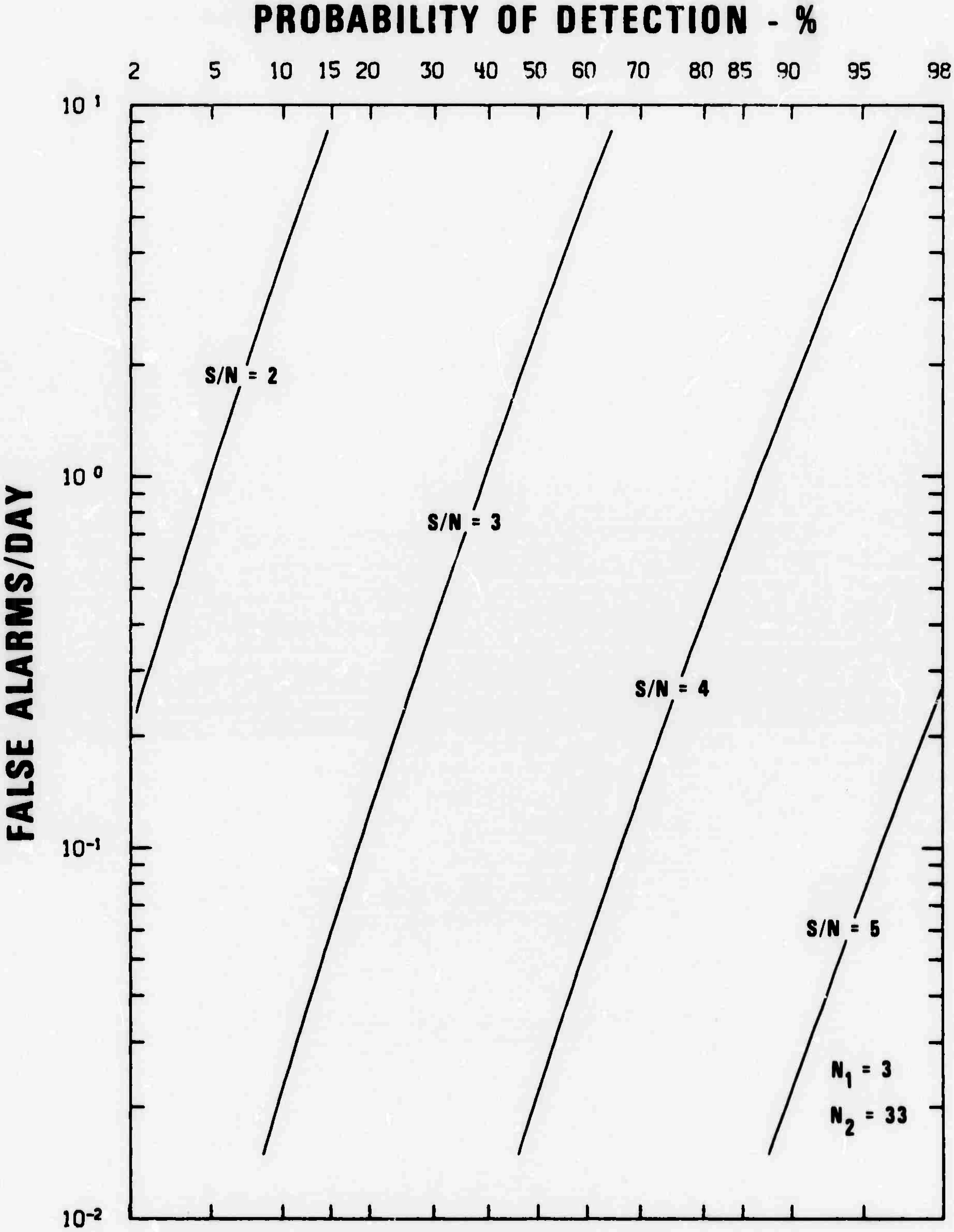

Figure 5. Probability of detection versus beam signal/noise, single subarray of 12 elements.

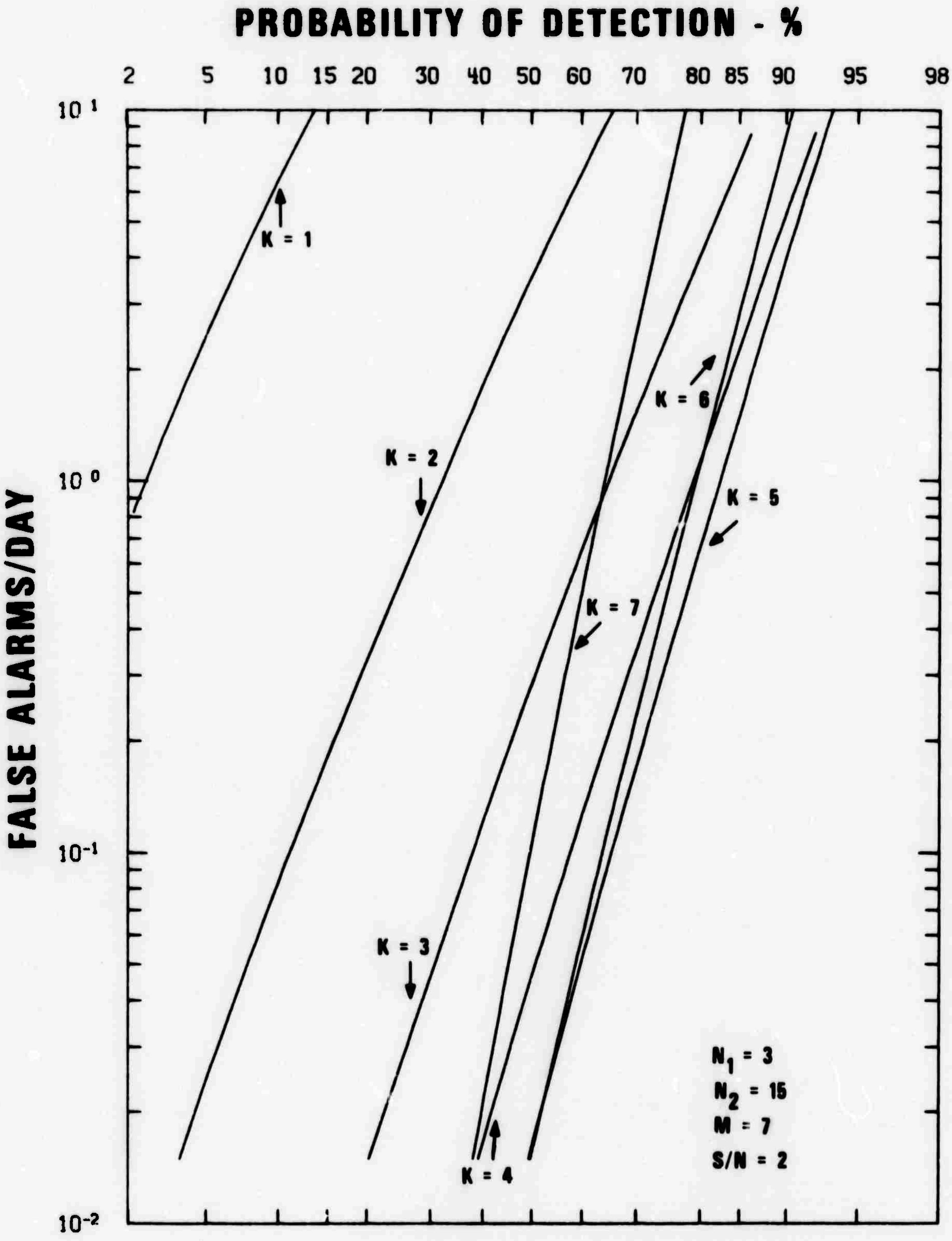

Figure 6. Probability of at least K subarrays out of 7 detecting (6 elements/subarray).

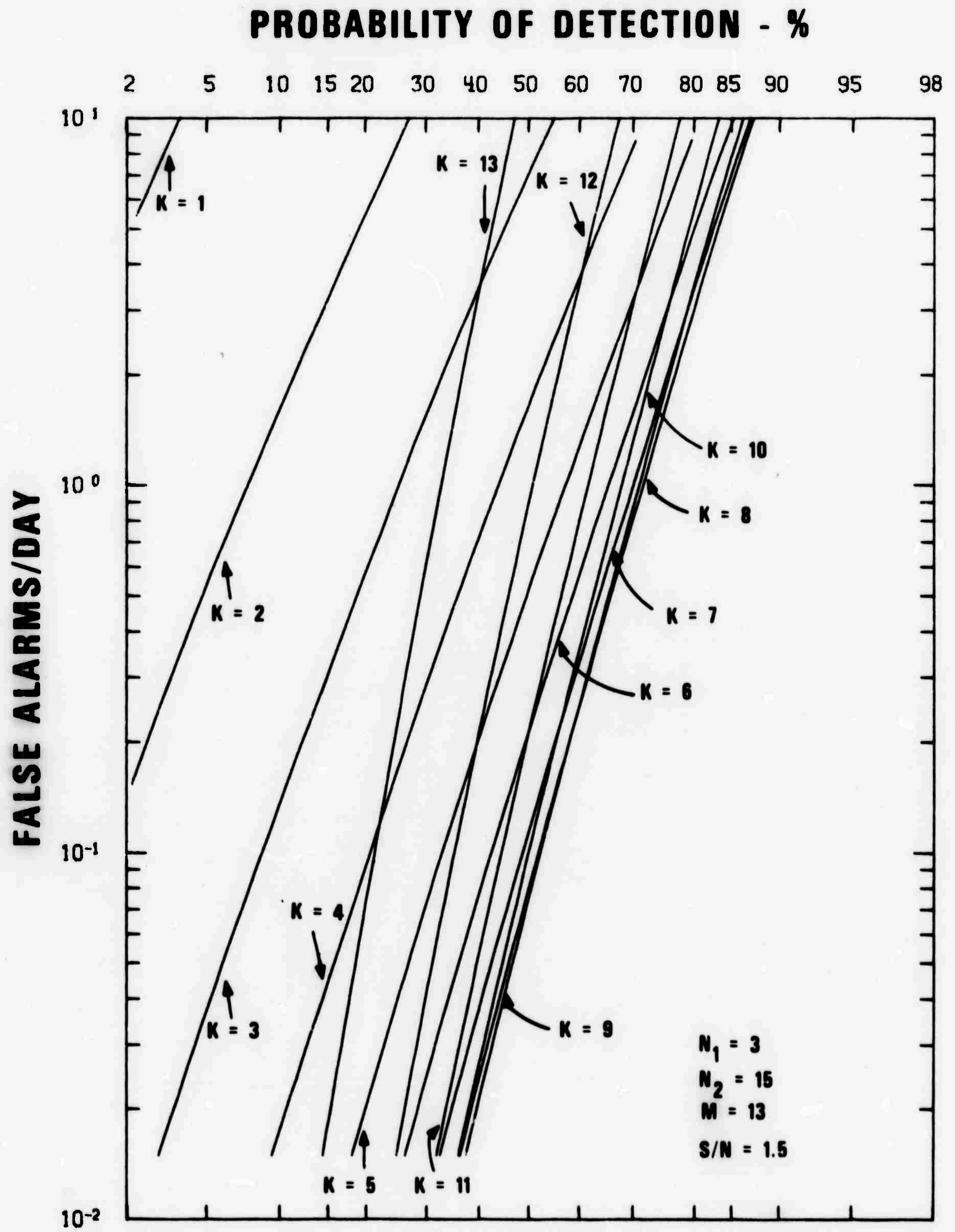

Figure 7. Probability of at least K subarrays out of 13 detecting (6 elements/subarray).

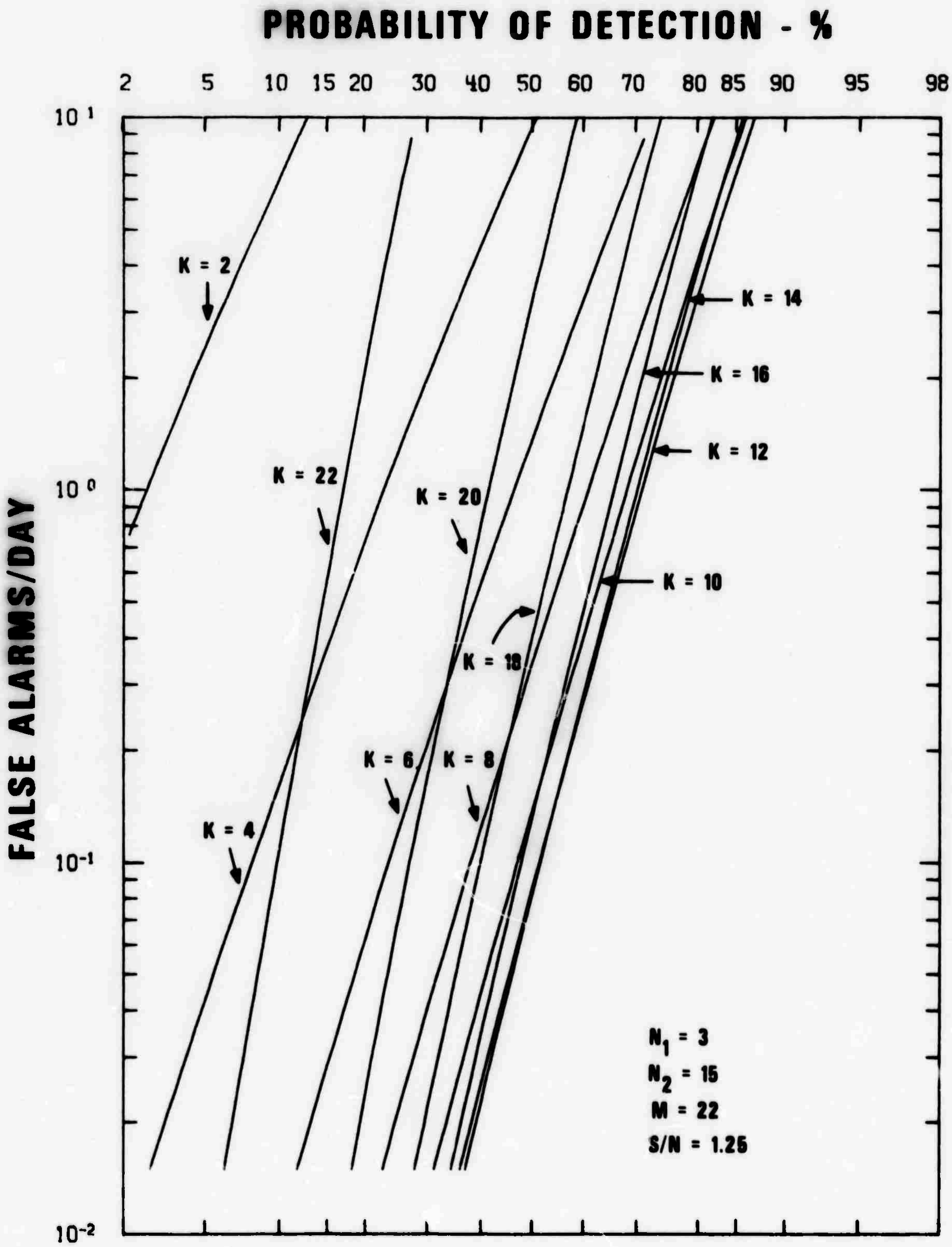

Figure 8. Probability of at least K subarrays out of 22 detecting (6 elements/subarray).

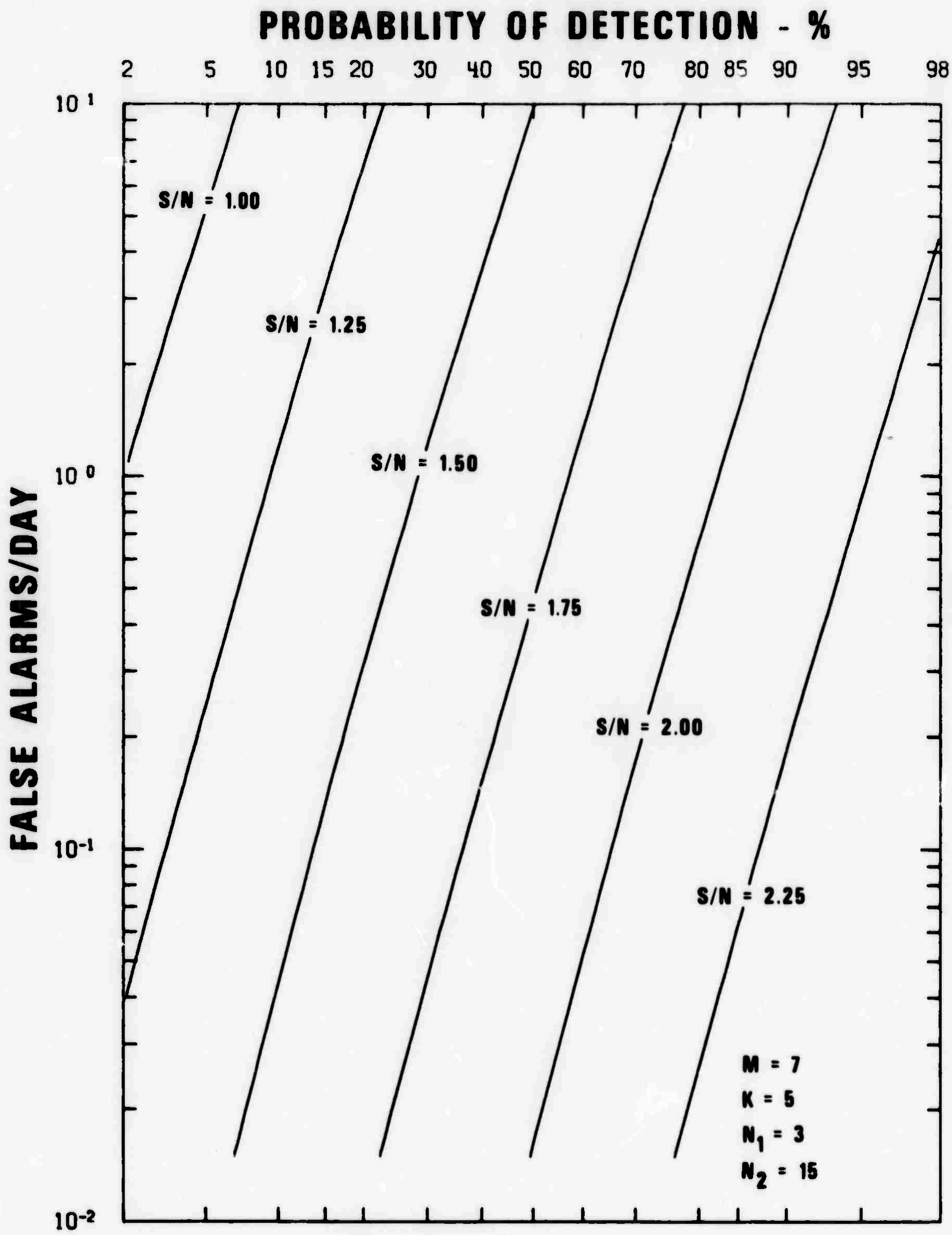

Figure 9. Probability of at least 5 subarrays out of 7 detecting versus beam signal/noise (K optimum, 6 elements/subarray).

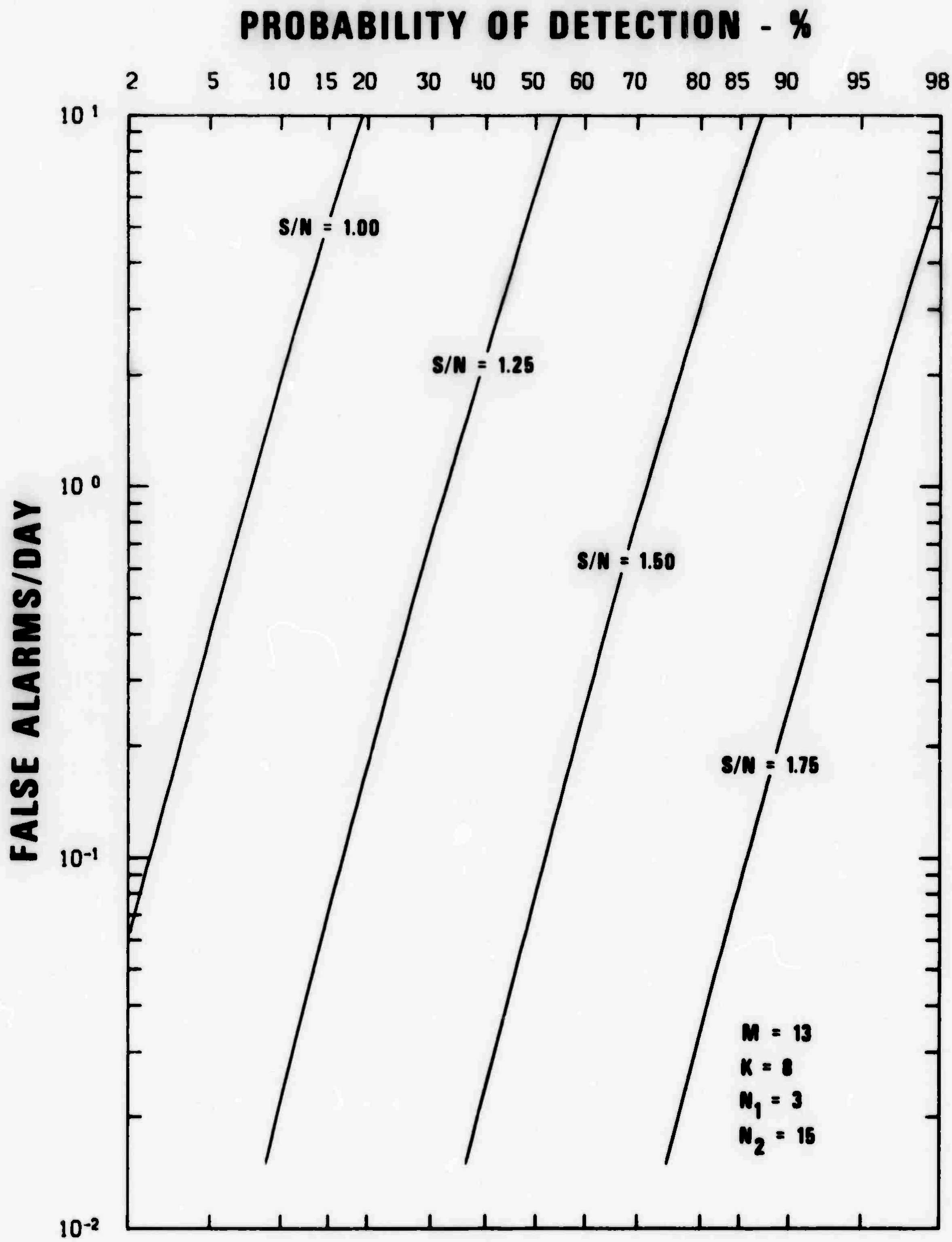

Figure 10. Probability of at least 8 subarrays out of 13 detecting versus beam signal/noise (K optimum, 6 elements/ subarray).

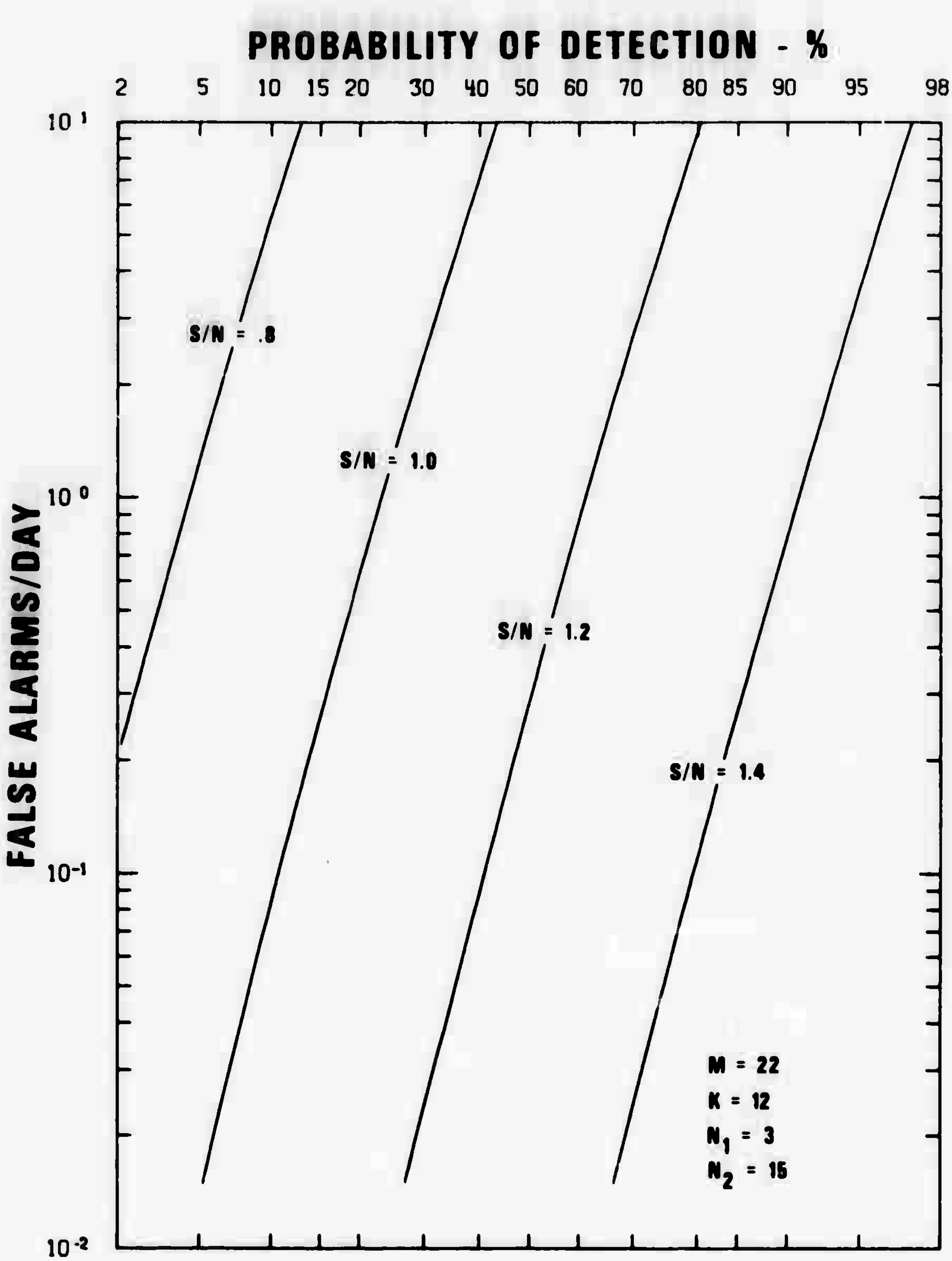

Figure 11. Probability of at least 12 subarrays out of 22 detecting versus beam signal/noise (K optimum, 6 elements/ subarray).

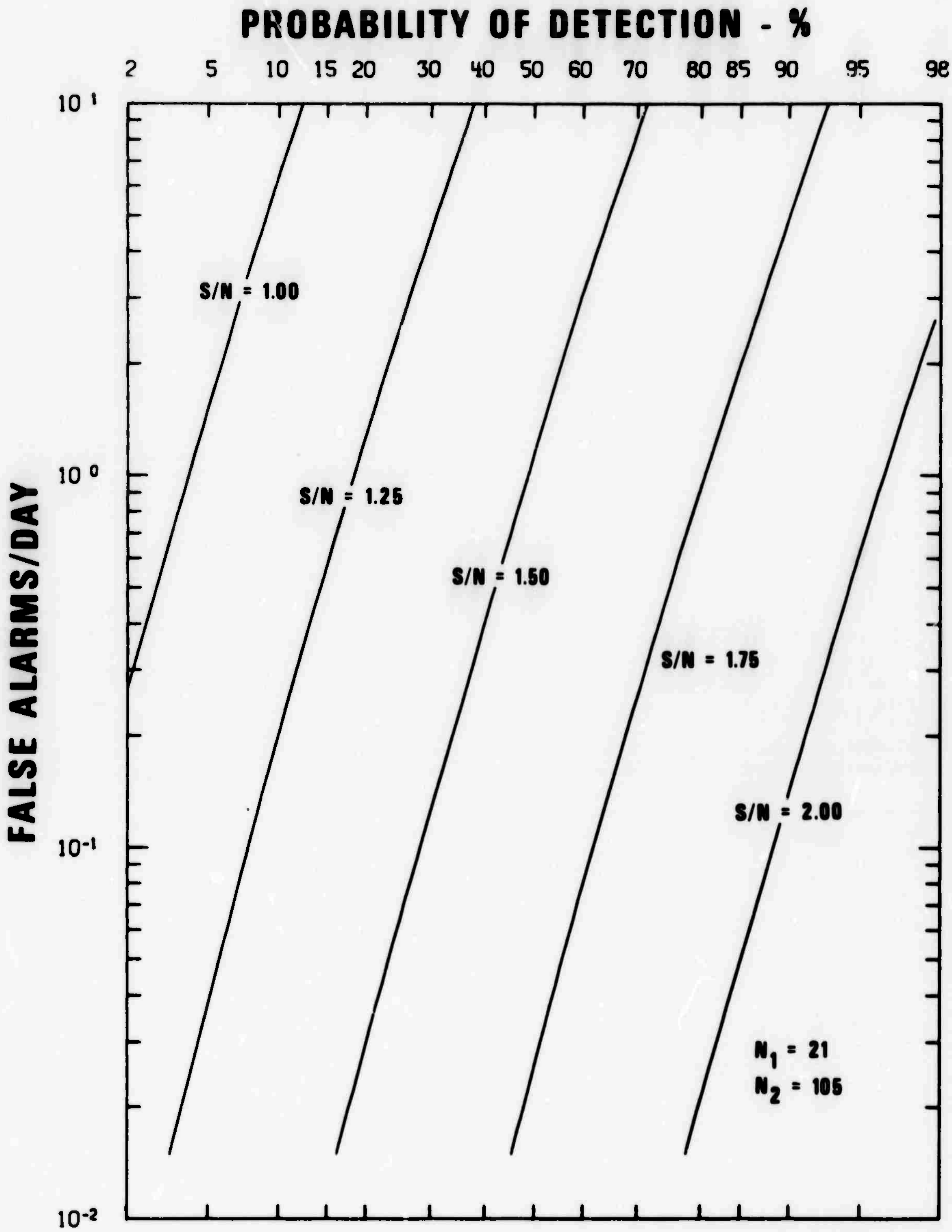

Figure 12. Probability of detection, multi-array F detector, versus subarray beam signal/noise (7 subarrays, 6 elements/ subarray).

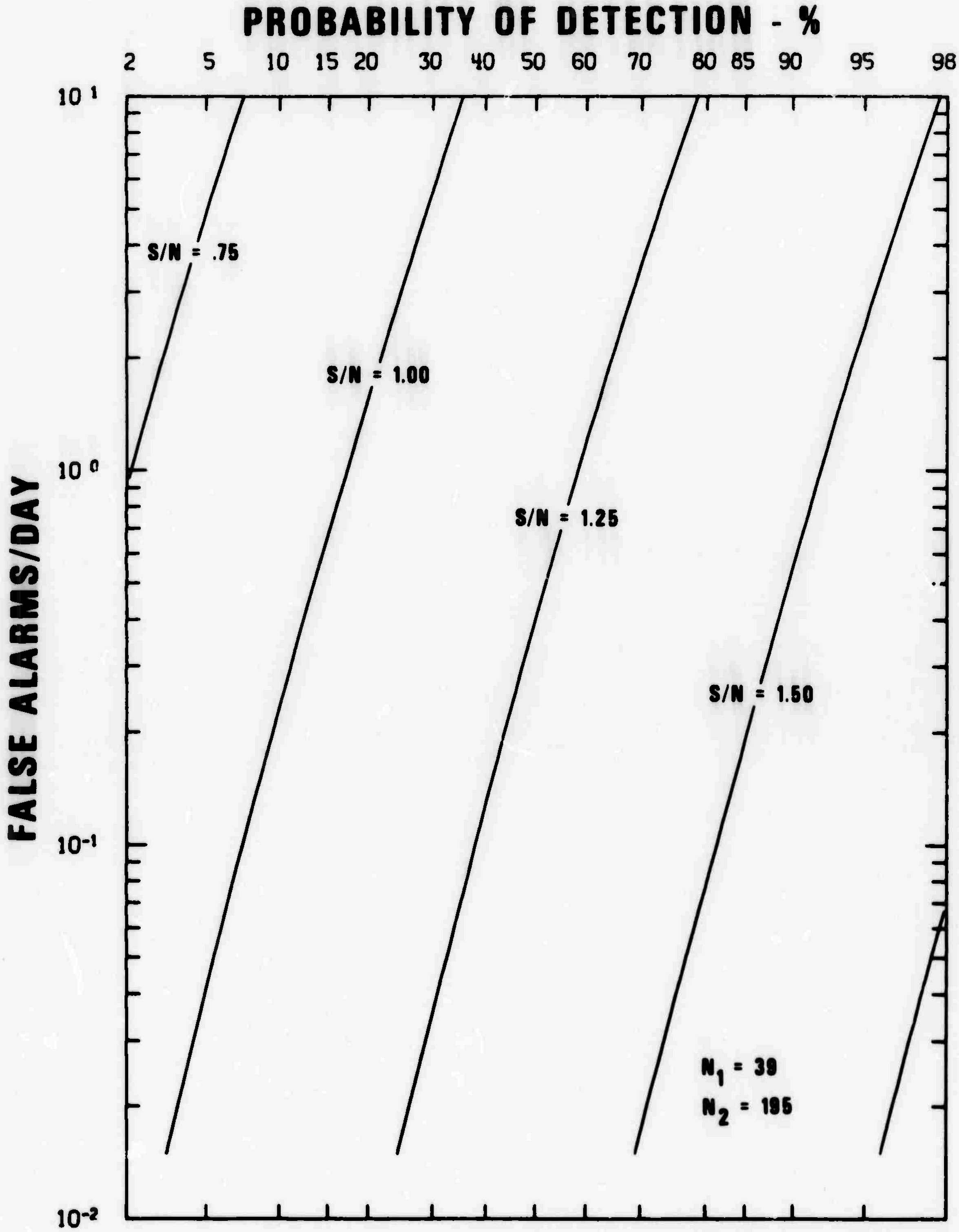

Figure 13. Probability of detection, multi-array F detector, versus subarray beam signal/noise (13 subarrays, 6 elements/subarray).

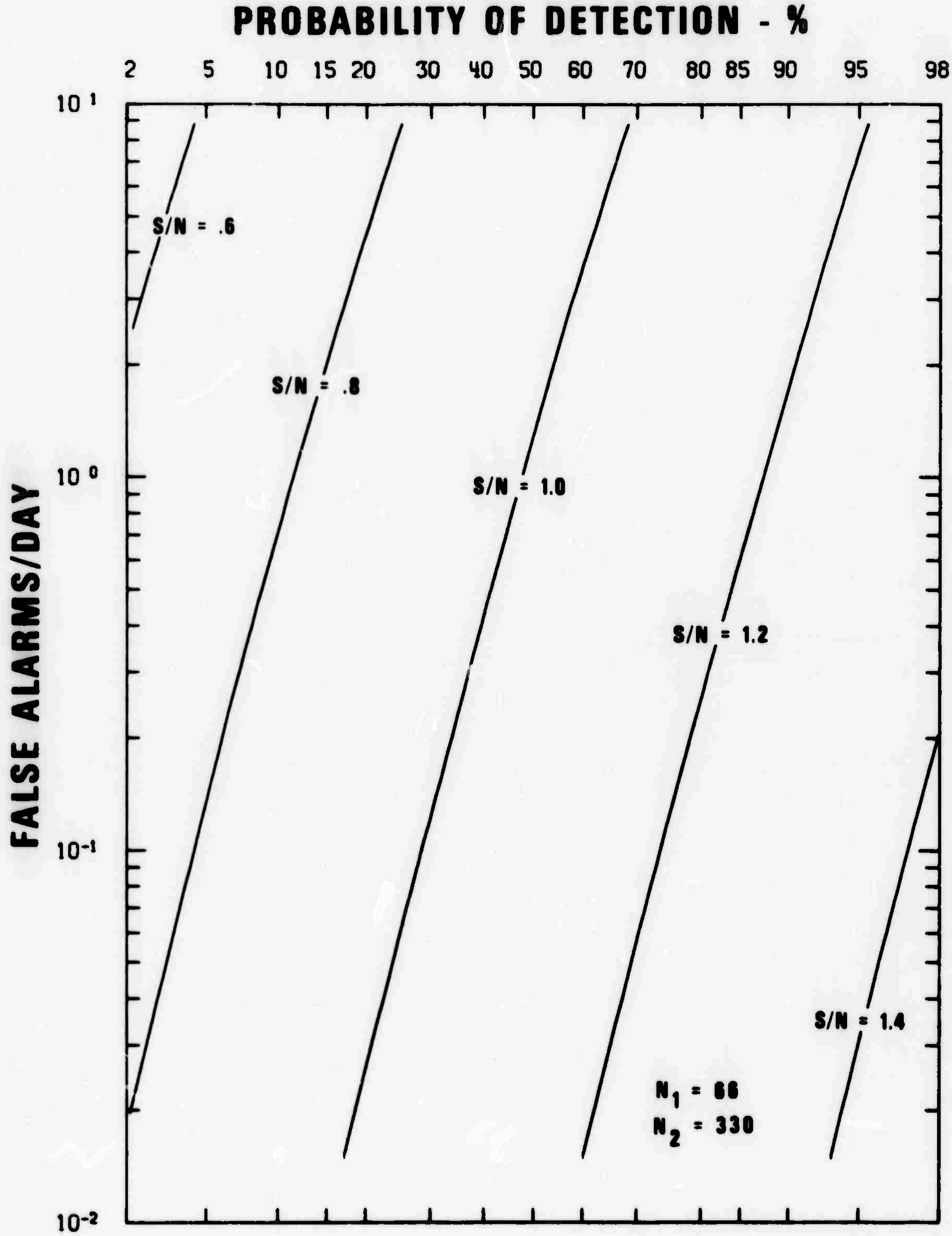

Figure 14. Probability of detection, multi-array F detector, versus subarray beam signal/noise (22 subarrays, 6 elements/subarray).

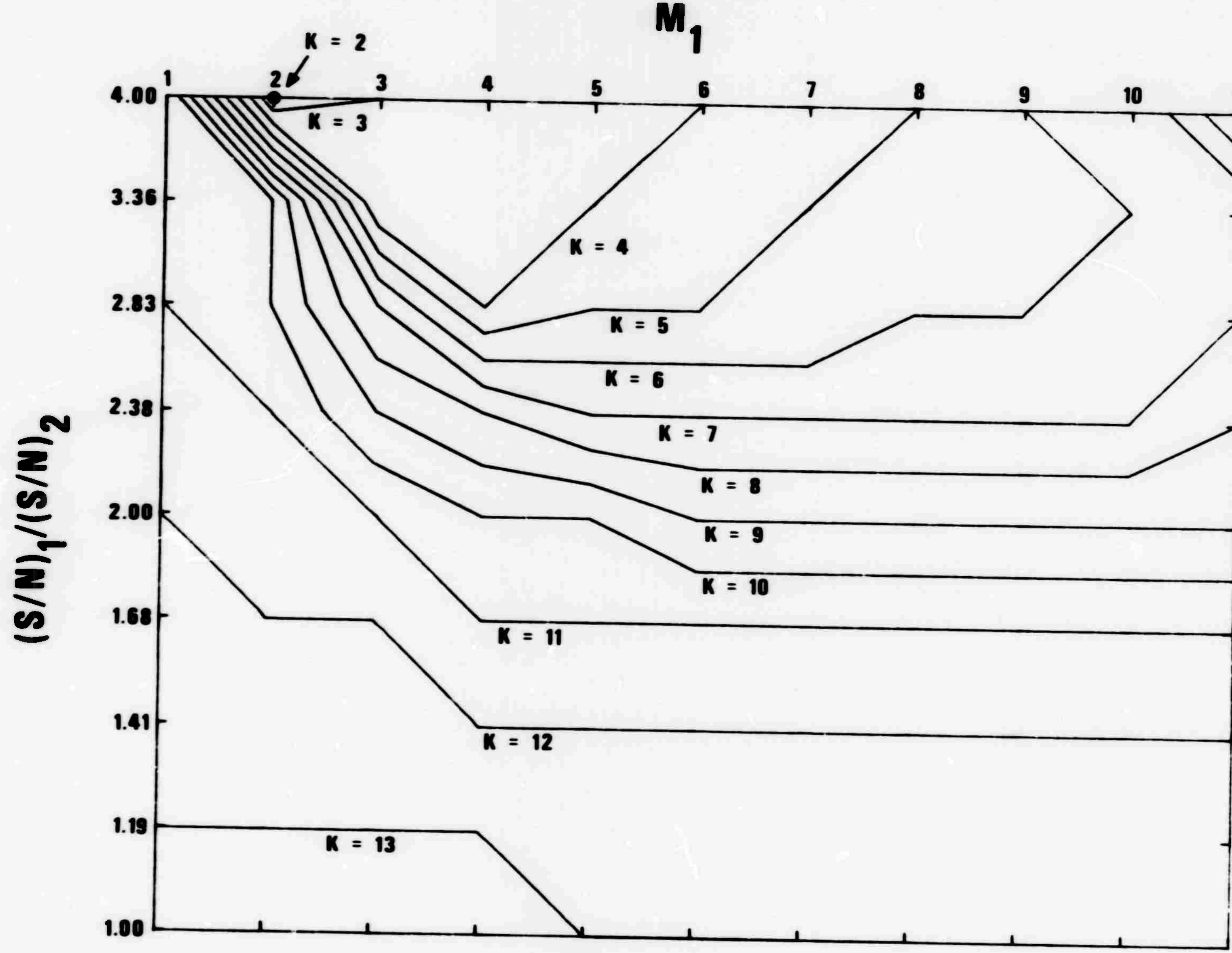

Figure 15. Contours of optimum K for network of 22 stations with a few strong stations, $(S/N)_2$ fixed (6 elements/array, 1 false alarm/day; $N_1 = 3$, $N_2 = 15$, $M_1+M_2 = 22$, $(S/N)_2 = 1$).

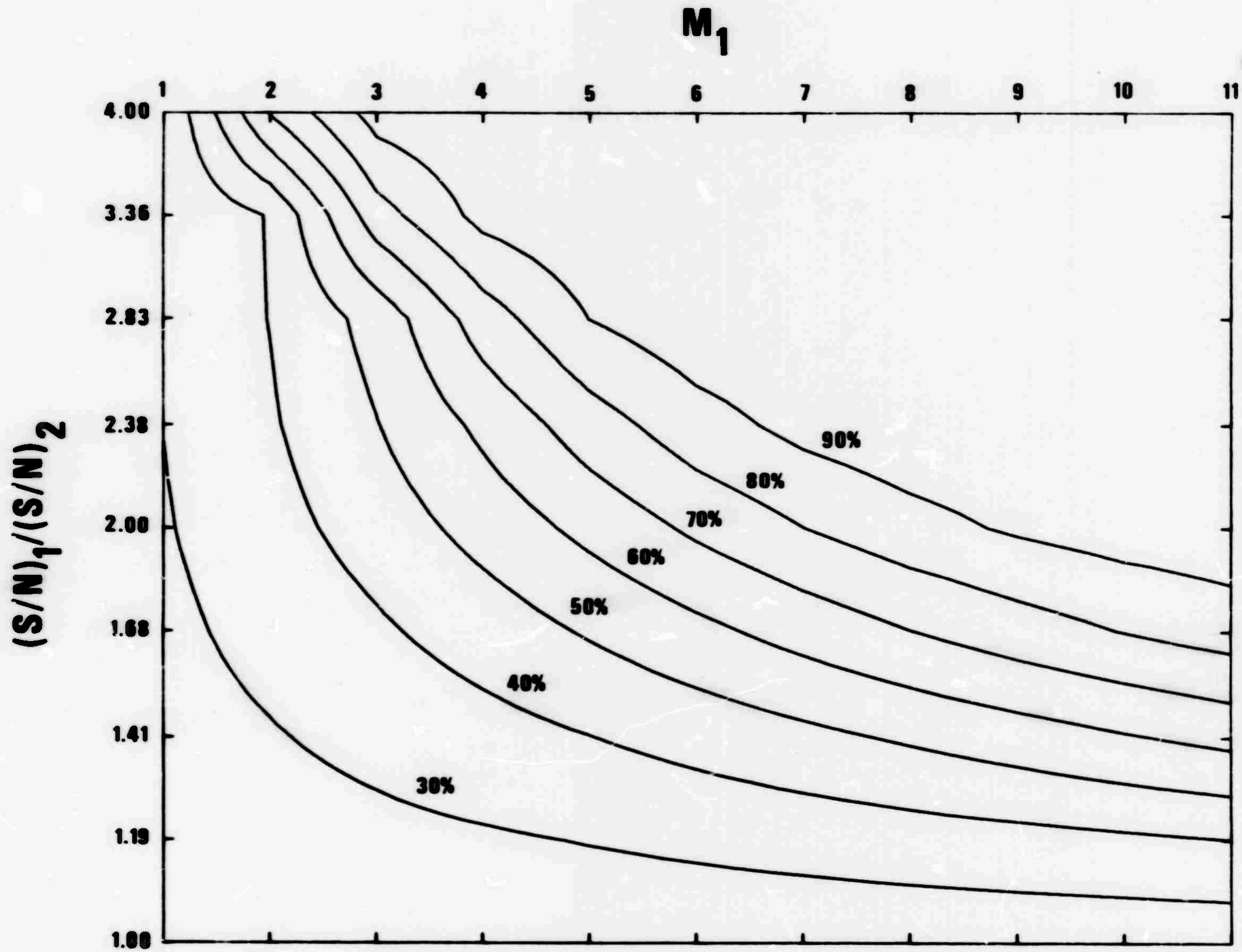

Figure 16. Contours of detection probability for a network of 22 stations with a few strong stations, $(S/N)_2$ fixed (K optimum, 6 elements/array, 1 false-alarm/day; $N_1 = 3$, $N_2 = 15$, $M_1+M_2 = 22$, $(S/N)_2 = 1$).

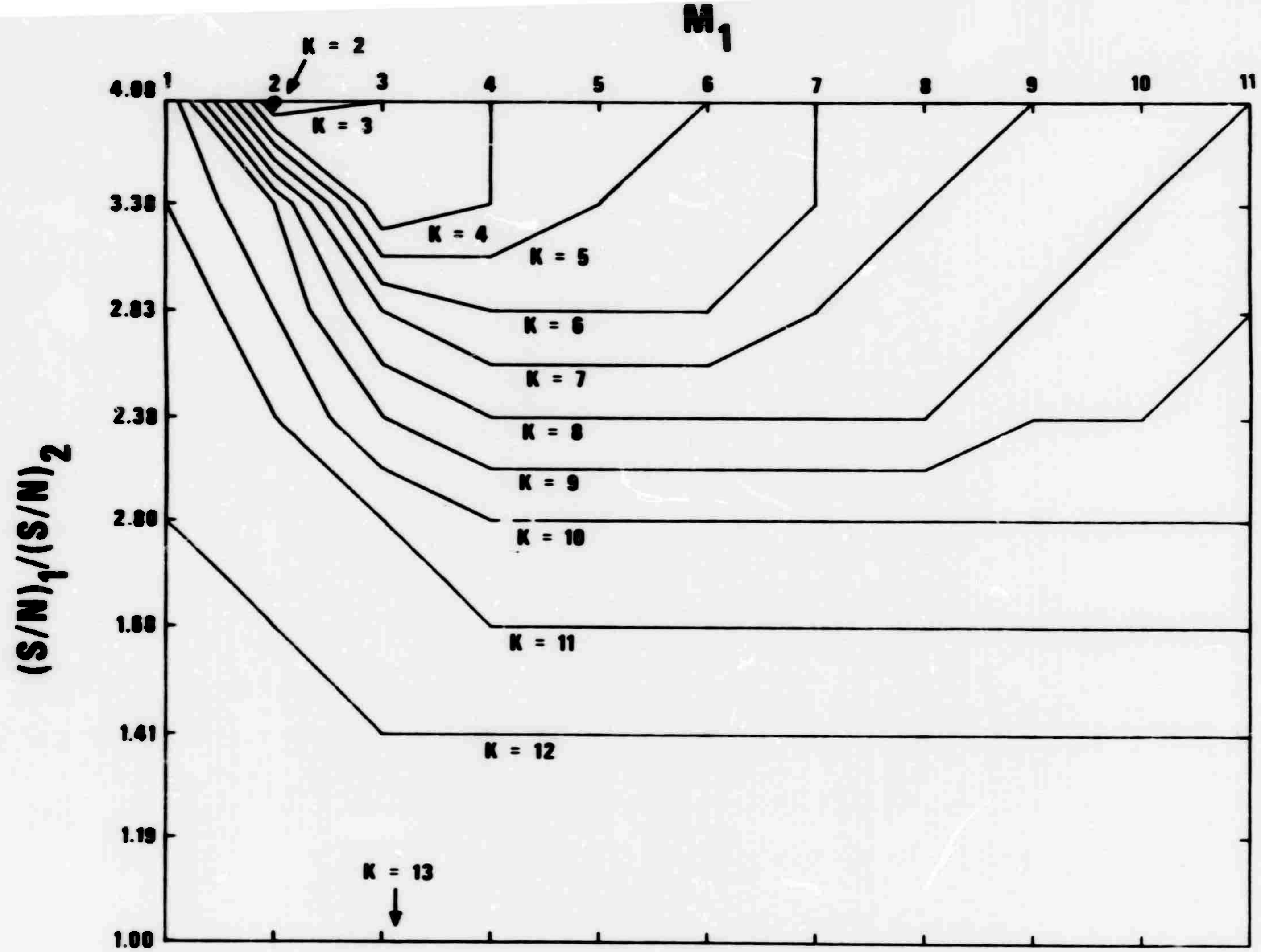

Figure 17. Contours of optimum K for a network of 22 stations with a few strong stations, rms S/N fixed (6 elements/array, 1 false alarm/day; $N_1 = 3$, $N_2 = 15$, $M_1+M_2 = 22$, $\overline{S|N} = 1.224$).

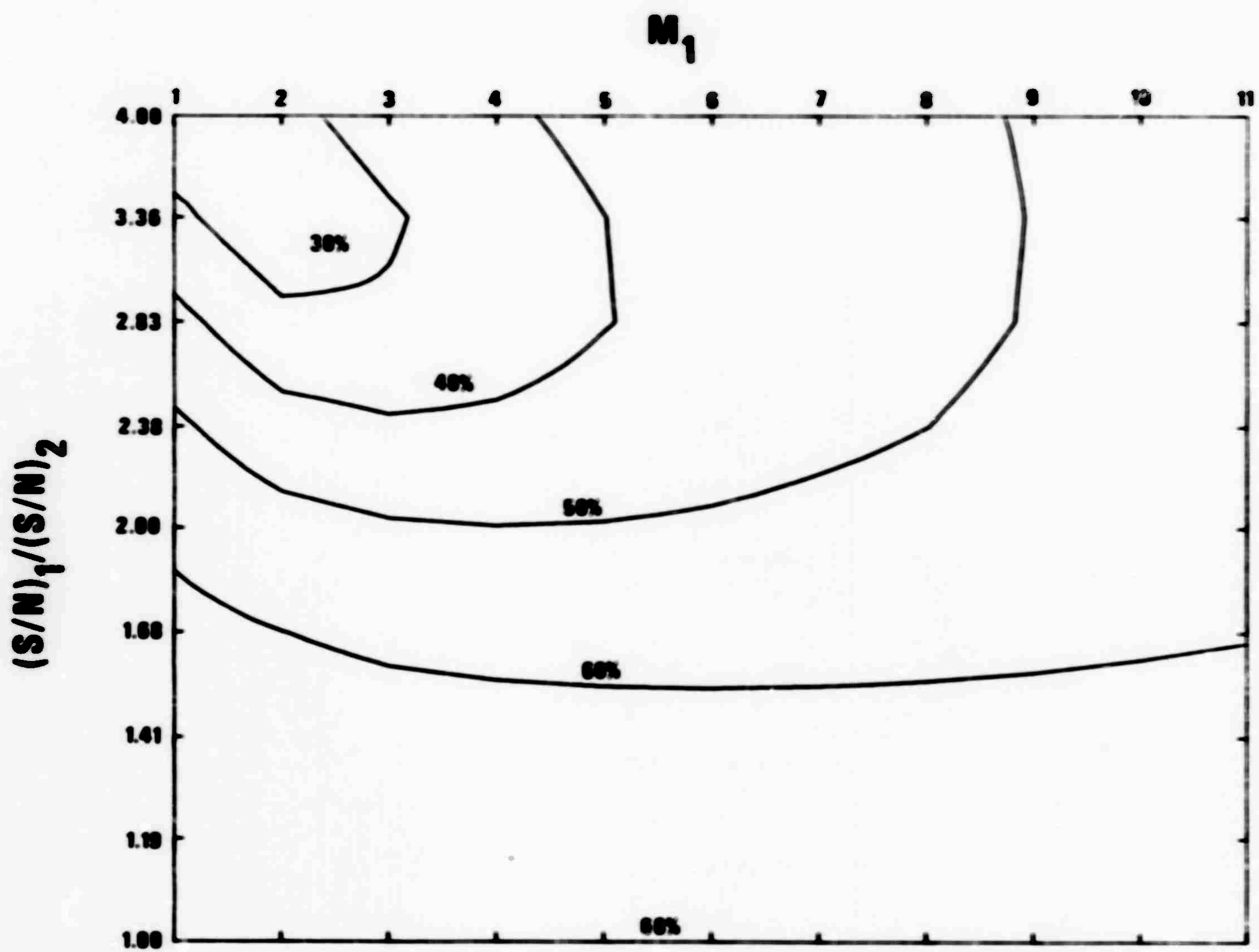

Figure 18. Contours of detection probability for a network of 22 stations with a few strong stations, rms S/N fixed (K optimum, 6 elements/array, 1 false-alarm/day; $N_1 = 3$, $N_2 = 15$, $M_1 + M_2 = 22$, $\overline{S/N} = 1.224$).

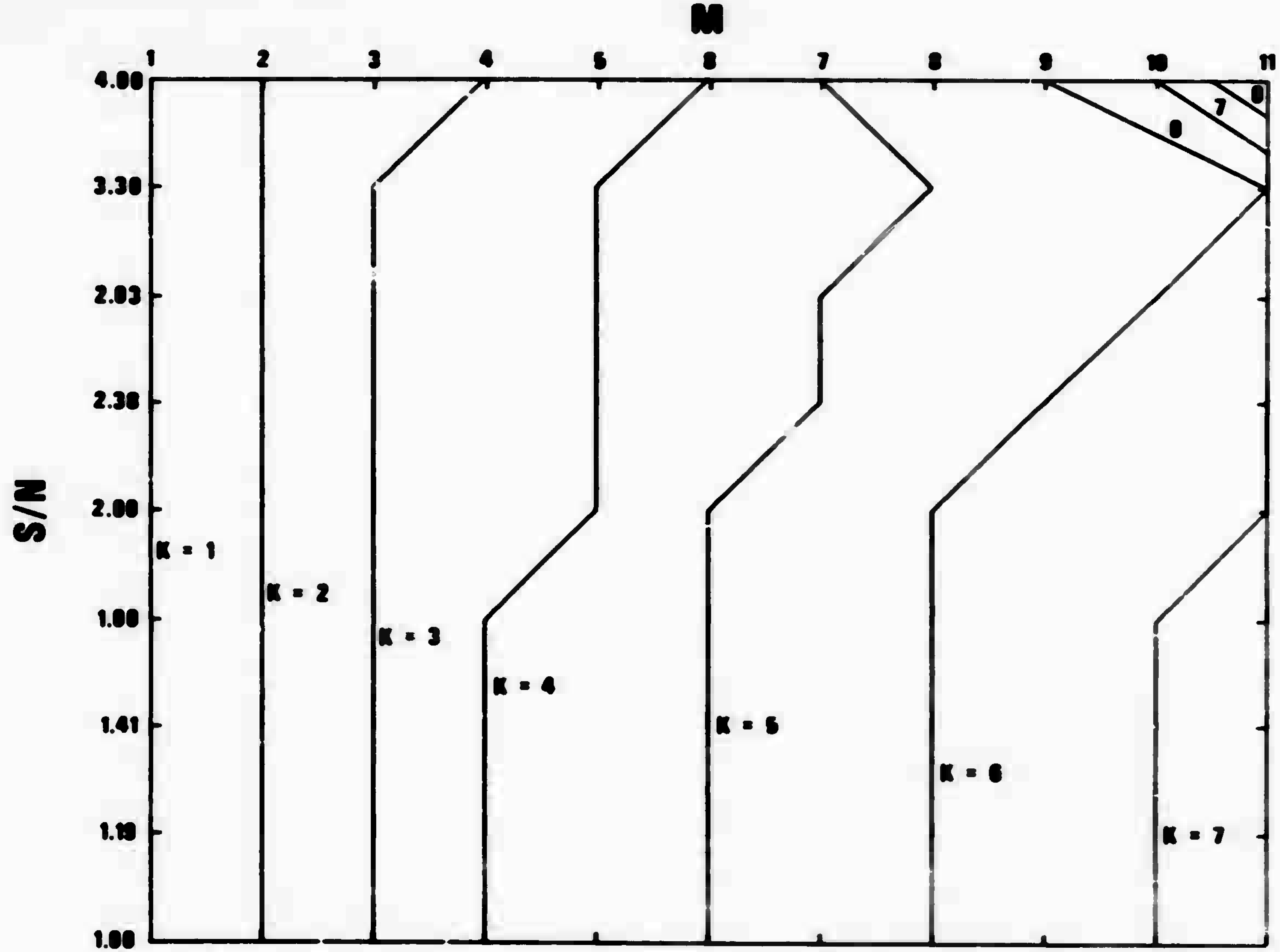

Figure 19. Contours of optimum K for a network of N stations (only the strong stations; 6 elements/array, 1 false-alarm/day; $N_1 = 3$, $N_2 = 15$).

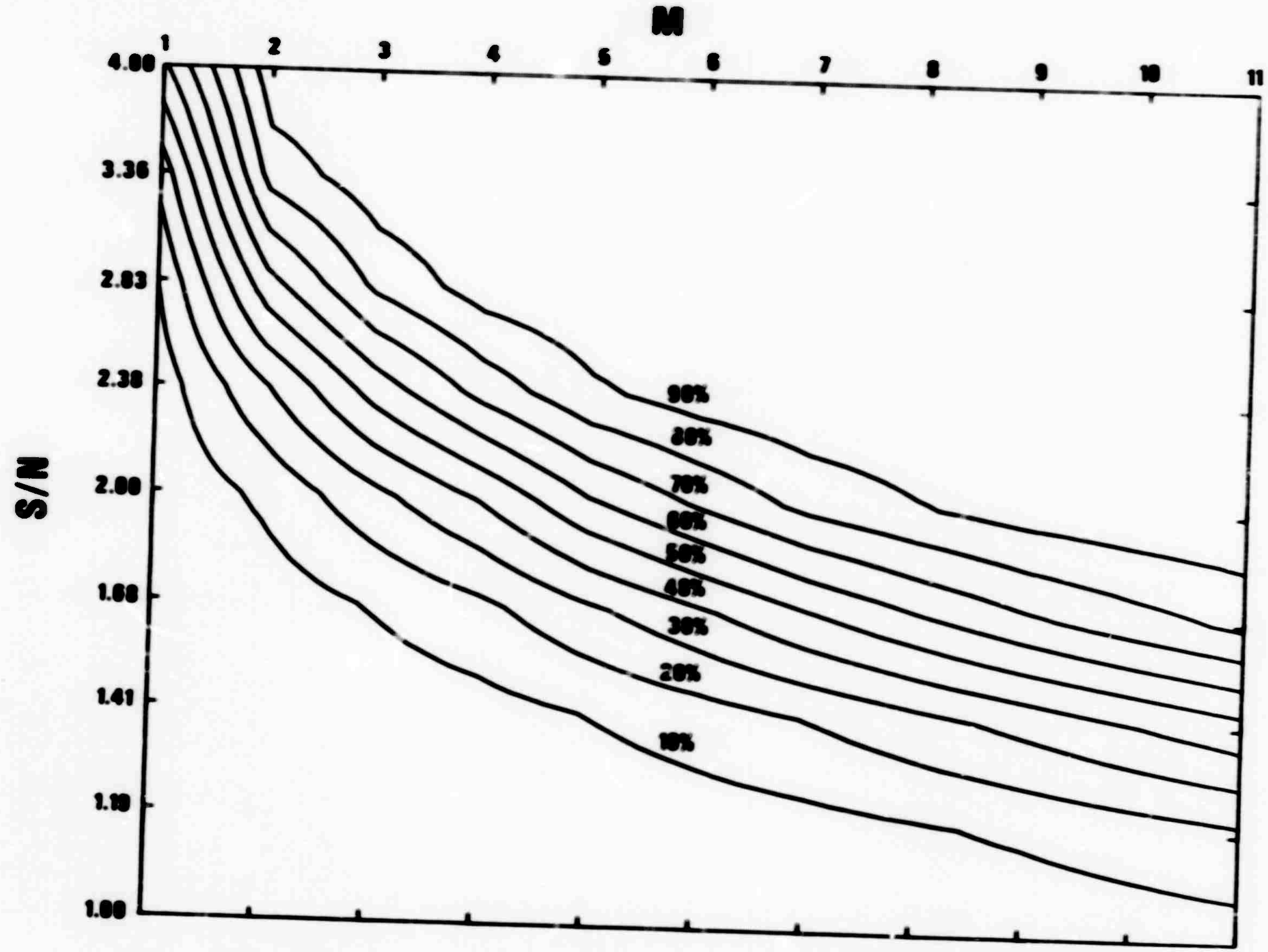

Figure 20. Contours of detection probability for a network of M stations (only the strong stations; K optimum; 6 elements/array, 1 false-alarm/day; $N_1 = 3$, $N_2 = 15$).

APPENDIX

STATISTICAL CALCULATIONS

NONCENTRAL F: Subroutines have been written to evaluate the cumulative distribution function of the noncentral F distribution for all parities of ν_1 and ν_2. The necessary formulas have been taken from Abramowitz and Stegun (1966), with corrections. Their formula 26.6.18, giving the relationship between the central and non-central distributions, is incorrect, as pointed out by Blandford (1970). The correct form is

$$P(F'|\nu_1,\nu_2,\lambda) = \sum_{j=0}^{\infty} e^{-\lambda/2} \frac{(\lambda/2)^j}{j!} P\left(\frac{F'\nu_1}{\nu_1+2j}\middle|\nu_1+2j,\nu_2\right) \qquad (1)$$

The upper integral $Q \equiv \text{prob}(u \geq F)$ is the one evaluated here, and the relationship is $Q = 1 - P$. It is easy to see that (1) still holds if the P's on both sides are replaced by Q's. This formula is used for all three cases. The sum always converges for finite λ, however, overflows may occur if $e^{\lambda/2}$ exceeds the range of floating-point variables. On the CDC 1604 computer, this restricts the noncentrality parameter to $\lambda \leq 1416$. The maximum of 2000 terms of (1) taken in the subroutines should be adequate within this range of λ for an accuracy of at least six significant figures in Q.

Observe that the third parameter (ν_2) of Q on the RHS of (1) does not depend on j. Use is made of this fact in choosing formulas for $Q(F|\nu_1,\nu_2)$ in order to minimize computations. It has been possible in two cases to obtain forms which require only ~M operations, where M is the number of terms taken in the infinite series, rather than

M^2 as might have been expected. In the third case, ν_2 even, the infinite sum can be done explicitly. These economies are possible because the parameter x in all the expansions for Q, defined by

$$x \equiv \frac{\nu_2}{\nu_2 + \nu_1 F} \tag{2}$$

does not depend on j.

ν_1 even: For this case, formula 26.6.4 is used for Q:

$$Q(F|\nu_1,\nu_2) = x^{\nu_2/2}\Big[1 + \frac{\nu_2}{2}(1-x) + \frac{\nu_2(\nu_2+2)}{2\cdot 4}(1-x)^2 + \cdots$$
$$+ \frac{\nu_2(\nu_2+2)\cdots(\nu_2+\nu_1-4)}{2\cdot 4\cdots(\nu_1-2)}(1-x)^{\nu_1/2-1}\Big] \tag{3}$$

where x is defined by (2). Since the only dependence on ν_1 (and hence on j) is in the number of terms, each additional term in the infinite series in (1) requires only one additional term in (3), supporting the claim made in the paragraph above. Listings of all the subroutines (QNCF) are given at the end of this section.

ν_1 and ν_2 odd: Formula 26.6.8 can, with some manipulation, be written in the more convenient form

$$Q(F|\nu_1,\nu_2) = \frac{2}{\pi} \{\phi - x \cot\phi \; [1 + \frac{2}{3} x + \cdots$$

$$+ \frac{2\cdot 4\cdots(\nu_2-3)}{3\cdot 5\cdots(\nu_2-2)} x^{(\nu_2-3)/2}]\}$$

$$+ \frac{2}{\pi}\cdot\frac{2\cdot 4\cdots(\nu_2-1)}{1\cdot 3\cdots(\nu_2-2)} x^{(\nu_2+1)/2} \cot\phi \; [1 + \frac{\nu_2+1}{3} (1-x) + \cdots \tag{4}$$

$$+ \frac{(\nu_2+1)(\nu_2+3)\cdots(\nu_1+\nu_2-4)}{3\cdot 5\cdots(\nu_1-2)} (1-x)^{(\nu_1-3)/2}]$$

where

$$\phi = \arctan\sqrt{\nu_2/\nu_1 F}$$

and x is defined by (2). Note that, once again, the only dependence on ν_1 is in the number of terms (of the second series), so that an efficient calculation is possible here also. Use is also made of the fact that the first series can be removed from the infinite sum since it is independent of j.

ν_2 even: Formula 26.6.22 is, unfortunately, incorrect, nor did several obvious modifications of the formula yield correct answers. A similar formula was derived by Wirth (1971), starting from 26.6.5 written in the form

$$P(F|\nu_1,\nu_2) = (1-x)^{\nu_1/2} [1 + \frac{\nu_1}{2} x + \frac{\nu_1(\nu_1+2)}{2\cdot 4} x^2 + \cdots$$

$$+ \frac{\nu_1(\nu_1+2)\cdots(\nu_1+\nu_2-4)}{2\cdot 4\cdots(\nu_2-2)} x^{\nu_2/2-1}] \tag{5}$$

The derivation will not be repeated here, but involves putting (5) into (1), interchanging the order of summations, applying Kummer's transform to the confluent hypergeometric series, and deriving a recursion for the resulting finite series. The result is a closed expression for Q which is very convenient for computation

$$Q(F|\nu_1,\nu_2,\lambda) = 1 - e^{-\lambda x/2} (1-x)^{\nu_1/2} \sum_{i=0}^{\nu_2/2-1} T_i \tag{6}$$

where

$$T_0 = 1$$

$$T_1 = \frac{x}{2} [\nu_1 + \lambda(1-x)]$$

$$T_i = \frac{x}{2i} \{[\nu_1+4i-4+\lambda(1-x)]T_{i-1} - (\nu_1+2i-4)xT_{i-2}\}$$

and x is defined by (2).

The three subroutines were checked against tables of the central F distribution in Abramowitz and Stegun (1966) with $\lambda = 0$. The routines for ν_1 even and ν_2 even were checked against each other and against routines which had been written independently by Blandford (1970) from formulas 26.6.6 and 26.6.7. In addition, all routines were checked against the Pearson and Hartley charts reproduced in Scheffe (1959), relating noncentrality parameters according to $\lambda = \phi^2(\nu_1+1)$. The agreement in all cases was within the limits of readability or table accuracy, or the normal limits of single-precision computation. The nominal accuracy of the first two routines is about six digits, due to the truncation of the infinite series.

INVERSE F: A very useful routine has also been developed for computing the inverse of the central F distribution function, $F(Q,\nu_1,\nu_2)$, which gives the threshold level corresponding to a given false-alarm rate. This routine has made possible the automatic plotting of detection probability directly in terms of false-alarm rate. Newton's iteration

$$F_{i+1} = F_i - \frac{Q(F_i) - Q}{Q'(F_i)} \tag{7}$$

is used, with the previously described routines (QNCF) being used to compute Q(F). The derivative is obtained from 26.6.1 by differentiation

$$Q'(F) = -\frac{(rF)^{\nu_1/2}}{FB(\tfrac{1}{2}\nu_1,\tfrac{1}{2}\nu_2)(1+rF)^{(\nu_1+\nu_2)/2}}$$

where $r \equiv \nu_1/\nu_2$ and B is the beta function: $B(a,b) \equiv \Gamma(a)\Gamma(b)/\Gamma(a+b)$. Starting values are derived from 26.6.16 and 26.5.22 over part of the range of arguments. The inverse of the normal cumulant required for this formula is provided by a routine (QUANTF) derived from a Hastings expansion, formula 26.2.23. Over the rest of the range, a starting value is obtained from the asymptotic form

$$Q(F|\nu_1,\nu_2) \approx \frac{2}{\nu_2 B\ (1+rF)^{\nu_2/2}} \quad , \quad \text{for } rF >> 1$$

from which

$$F \approx \frac{1}{r}\ [(\frac{2}{\nu_2 BQ})^{2/\nu_2} - 1]$$

where arguments of B are omitted for simplicity. (This formula is exact for $\nu_1 = 2$.) The resulting routine (FINV) converges quite well over a broad range of input parameters. In less than 10 iterations it will match the input value of Q to at least 5 digits relative accuracy. (Note, however, that the second two versions of QNCF have a lower limit on absolute precision of $\sim 10^{-10}$ due to rounding. Thus the relative accuracy will be reduced for $Q << 10^{-5}$.)

An inverse of the noncentral F might also be useful, $\lambda(F,Q,\nu_1,\nu_2)$, giving the noncentrality parameter in terms of the probability, i.e. the signal/noise ratio required for a given probability of detection. The derivative for a Newton's iteration (7) is easy to obtain from (1):

$$\frac{\partial Q(F|\nu_1,\nu_2,\lambda)}{\partial\lambda} = \frac{1}{2}\left[Q\left(\frac{F\nu_1}{\nu_1+2}\middle|\nu_1+2,\nu_2,\lambda\right) - Q(F|\nu_1,\nu_2,\lambda)\right]$$

but it is not so easy to see how to derive a starting value. The approximations in Abramow' z and Stegun are impractical for this purpose. Actually, such a routine has not been needed here. Signal/noise thresholds quoted in the text were obtained graphically by linear interpolation with respect to (S/N). The high linearity of the curves with respect to both (S/N) and false-alarm rate may furnish a clue to obtaining a starting value for the automatic calculation, should this approach seem desirable in the future.

BINOMIAL: The routine for computing the summed binomial distribution, equation (1) of the text, is completely straightforward. A listing of the subroutine (POFK) is given below.

INVERSE BINOMIAL: The inverse of the binomial distribution is used with the inverse of the F distribution to obtain the threshold in terms of the false-alarm rate for the composite ($\geq$K) detector. A Newton's iteration (7) is used with a modified derivative. The derivative of P($\geq$k) is easily obtained from (1) of the text

$$\frac{dP(\geq k)}{dp} = k \binom{M}{k} p^{k-1}(1-p)^{M-k} \tag{8}$$

and it is easily computed, however, the iteration did not converge as well as might have been expected when this derivative was used with a starting value derived from an approximation to $P(\geq k)$. It is easily shown that the lowest term of $P(\geq k)$ is the largest if $p < (k+1)/(M+1)$, so that

$$P(\geq k) \approx \binom{M}{k} p^k (1-p)^{M-k}$$

from which a starting value $p \approx [P(\geq k)/\binom{M}{k}]^{1/k}$ has been taken (assuming $p << 1$). Comparison of (8) and (9) shows that

$$\frac{dP(\geq k)}{dp} \approx \frac{k}{p} P(\geq k)$$

Use of this approximation to the derivative gave much improved convergence with the above starting value. The routine (POFKINV) is quite simple and converges well over a broad range, even when the condition for (9) does not hold. In less than 15 iterations it will match the input value of $P(\geq k)$ to at least 5 digits relative accuracy. No trouble was experienced, but if any should be, (9) could be replaced with the term actually biggest, which is just (9) with k replaced by the smallest integer $\geq p(M + 1)-1$, which might be done recursively. The derivative might also need to be changed in that case.

OTHER CALCULATIONS: Routines were written to compute equation (2) of the text and its inverse, but they are not included here because of the negative results and their doubtful utility. The inverse routine was not very satisfactory and frequently failed to converge, although more than enough highly accurate results were obtained to reach the conclusions stated in the text.

A special routine was written to compute the probability of at least k events out of two groups having different probabilities, although the more general procedure of Wirth (1971) could also have been used. In the notation of (2)

$$P(\geq k) = \sum_{i} P\left(\tbinom{M_1}{i} \middle| P_1\right) P\left(\tbinom{M_2}{\geq(k-i)} \middle| p_2\right)$$

or

$$P(\geq k) = \sum_{i=i_s}^{M_1} \binom{M_1}{i} p_1^i (1-p_1)^{M_1-i} S_2$$

where

$$S_2 = \begin{cases} \sum_{j=k-i}^{M_2} \binom{M_2}{j} p_2^j (1-p_2)^{M_2-j} , & \text{if } i<k \\ 1 , & \text{if } i \geq k \end{cases}$$

and $i_s \equiv \max(0, k-M_2)$. The routine (POFK3) is fairly straightforward. False-alarm rates for this case were computed from POFK and its inverse, since false-alarm rates were assumed to be the same for all stations.

```
                                                              06 09 71
      FUNCTION QNCF( F,EL,N1,N2 )                                       N1 EVEN
C     QNCF = PROB( U>F ), WHERE U DISTRIB. AS NON-CENTRAL F, W/ NON-
C     CENTRALITY PARAM. *EL* AND *N1,N2* DEGREES OF FREEDOM, (N1 EVEN).
C
      EL2 = EL *.5          $     EN1 = N1          $     EN2 = N2
      EM = EN2 *.5          $     X = EN2/(EN2 + EN1*F)
      Y = 1.- X             $     SM = T = X**EM
      FI = 1.               $     NUT = N1/2 - 1
      DO 10 I = 1,NUT
      T = T * Y * EM/FI     $     SM = SM + T       $     EM = EM + 1.
   10 FI = FI + 1.
      P = SM                $     AJ = EL2          $     D = 1.
      DO 20 J = 1,2000
      T = T * Y * EM/FI     $     SM = SM + T
      TJ = AJ * SM          $     P = P + TJ
      IF( TJ/P.LT.1E-7 ) 30,15
   15 EM = EM + 1.          $     FI = FI + 1.      $     D = D + 1.
   20 AJ = AJ * EL2/D
      PRINT 1
    1 FORMAT(/ 22H POOR CONVERGENCE QNCF)
   30 QNCF = P*EXPF(-EL2)
      RETURN
      END
```

```
      FUNCTION QNCF( F,EL,N1,N2 )                                       N1,2 ODD
      DATA (TPI = .63661977237)
C     QNCF = PROB( U>F ), WHERE U DISTRIB. AS NON-CENTRAL F, W/ NON-
C     CENTRALITY PARAM. *EL* AND *N1,N2* DEGREES OF FREEDOM, (N1,2 ODD).
C                                                                       MW
      EL2 = EL *.5          $     EN1 = N1          $     EN2 = N2
      X = EN2/(EN2 + EN1*F)                         $     Y = 1.- X
      TN = SQRTF( EN1*F/EN2 )                       $     THET = ATANF(1./TN)
      T = SM = 1.           $     FI = 3.           $     NUT = (N2-3)/2
      DO 10 I = 1,NUT
      T = T * X * (FI-1.)/FI                        $     SM = SM + T
   10 FI = FI + 2.
      A = TPI * (THET - X*TN*SM)
      G = TPI * T * (FI-1.) * X*X*TN                $     EM = EN2 + 1.
      T = SM = 1.           $     FI = 3.           $     NUT = (N1-3)/2
      DO 20 I = 1,NUT
      T = T * Y * EM/FI     $     SM = SM + T       $     EM = EM + 2.
   20 FI = FI + 2.
      P = SM                $     AJ = EL2          $     D = 1.
      DO 30 J = 1,2000
      T = T * Y * EM/FI     $     SM = SM + T
      TJ = AJ * SM          $     P = P + TJ
      IF( TJ/P.LT.1E-7 ) 40,25
   25 EM = EM + 2.          $     FI = FI + 2.      $     D = D + 1.
   30 AJ = AJ * EL2/D
      PRINT 1
    1 FORMAT(/ 22H POOR CONVERGENCE QNCF)
   40 QNCF = A + EXPF(-EL2)*G*P
      RETURN
      END
```

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```
      FUNCTION QNCF( F,EL,N1,N2 )                                   N2 EVEN
C     QNCF = PROB( U>F ), WHERE U DISTRIB. AS NON-CENTRAL F, W/ NON-
C     CENTRALITY PARAM. *EL* AND *N1,N2* DEGREES OF FREEDOM, (N2 EVEN).
C                                                                 MW
      EN1 = N1          $     FN2 = N2
      X = FN2/(FN2 + EN1*F)              $     Y = 1.- X
      EN14 = EN1 - 4.   $     ENLY = EN14 + EL*Y
      T2 = 0.           $     SM = T1 = 1.   $     F2I = 2.
      NUT = N2/2 - 1
      DO 10 I = 1,NUT
      TS = T1
      T1 = ((ENLY+2.*F2I)*T1 - (EN14+F2I)*X*T2) * X/F2I
      SM = SM + T1      $     T2 = TS
   10 F2I = F2I + 2.
      QNCF = 1.- EXPF((EN1*LOGF(Y) - EL*X)*.5) * SM
      RETURN
      END
```

```
      FUNCTION FINV( Q,N1,N2 )                                          ALL N1,N2
C     Q = PROB( U>FINV ), U DISTRIB. AS F, W/ N1,N2 DEGREES OF FREEDOM.
C     INVERSE CUMULANT OF F-DISTRIB., GIVES THRESHOLD AS A FUNC. OF
C     FALSE-ALARM RATE FOR FISHER DETECTOR.  (ALL N1,N2)               MW
C
      EN1 = N1              $     EN2 = N2            $    R = EN1/EN2
      EM = (EN1+EN2)*.5     $     EN1H = EN1*.5
      IF( XMODF(N1,2) ) 2,1
    1 D = EN2 *.5           $     U = B = 1.          $    NUT = N1/2 - 1
      GO TO  4
    2 IF( XMODF(N2,2) ) 6,3
    3 D = EN1 *.5           $     U = B = 1.          $    NUT = N2/2 - 1
    4 DO 5 I = 1,NUT
      B = B * U/D           $     U = U + 1.
    5 D = D + 1.
      B = B / D             $     GO TO  10
    6 B = 3.14159265358     $     U = .5              $    D = 1.
      IT = (N1-1)/2         $     NUT = (N1+N2)/2 - 1
      DO 9 I = 1,NUT
      B = B * U/D
      IF( I.EQ.IT ) 7,8
    7 U = -.5
    8 U = U + 1.
    9 D = D + 1.
   10 EN2R = 2./ EN2        $     RF = (EN2R/(B*Q))**EN2R
      IF( RF.GT.10. .OR. N1.LT.4 .OR. (N1.LT.6 .AND. Q.LT.1E-7) ) 15,20
   15 F = (RF-1.)/R         $     GO TO  30
   20 Y = -QUANTF( Q )      $     EL = Y*Y/6. -.5
      EN1 = 1./(N1-1)       $     EN2 = 1./(N2-1)     $    H = 2./(EN1+EN2)
      W = Y*SQRTF(H+EL)/H - (EN1-EN2)*(EL +.83333333333 -.66666666667/H)
      F = EXPF( 2.*W )
   30 DO 50 I = 1,10
      DQ = CNCF( F,0.,N1,N2 ) - Q
      IF( ABSF(DQ)/Q.LT.1E-5 ) 100,40
   40 RF = R * F
      DFDQ = B * (1.+RF)**EM / RF**EN1H
   50 F = F * (1.+ DFDQ*DQ)
  100 FINV = F
      RETURN
      END
```

```
      FUNCTION QUANTF(P)
C     P = PROB( U.LE.QUANTF ), WHERE U DISTRIB. AS ZERO-MEAN UNIT NORMAL.
C     INVERSE OF PROBF                                                  MW
      SIGN = 1.   $    Q = P
      IF(P.LE.0..OR P.GE.1.) 10,20
   10 QUANTF = P
      PRINT 1, P  $    RETURN
    1 FORMAT(24H ERROR IN QUANTF(P),  P= E20.10)
   20 IF( P.GT.0.5 ) 25,30
   25 SIGN = -1.  $    Q = 1.- P
   30 T = SQRTF(-2.* LOGF(Q))
      QUANTF = SIGN*( (2.515517 + T*(.802853 + T*.010328))/(1.+ T*
     *     (1.432788 + T*(.189269 + T*.001308))) - T )
      RETURN
      END
```

```
      FUNCTION POFK( N,K,P )
C     SUM OF BINOMIAL DISTRIB.
C     POFK = PROB. OF AT LEAST  K  EVENTS OUT OF  N
C     WHEN ALL EVENTS HAVE PROB.  P
C
      POFK = T = P**N                                              MW
      R = (1.-P) / P     $    A = N           $    B = 1.
      KP = K + 1
      DO 10 I = KP,N
      T = T * (A/B) * R
      A = A - 1.         $    B = B + 1.
   10 POFK = POFK + T
      RETURN
      END
```

```
      FUNCTION POFKINV( N,K,P )
C     INVERSE OF POFK
C     P = PROB. OF ≥K EVENTS WHEN ALL HAVE PROB. *POFKINV*
C
      C = A = 1.         $    B = N                                MW
      DO 5 I = 1,K
      C = C * A/B        $    A = A + 1.
    5 B = B - 1.
      RK = 1./ K
      PP = (P*C)**RK
      DO 10 I = 1,15
      DP = POFK( N,K,PP )/P - 1.
      IF( ABSF(DP).LT.1E-5 ) 20,10
   10 PP = PP * (1.- DP*RK)
   20 POFKINV = PP
      RETURN
      END
```

```
      FUNCTION POFK3( K,N1,N2, P1,P2 )
C     PROB. OF ≥K TOTAL OF TWO SETS, THE FIRST CONTAINING N1 ELEMENTS
C     W/ PROB. P1, THE SECOND N2 ELEMENTS W/ PROB. P2
C     N2 ≥ N1  AND  K ≤ N1+N2
C
      T1 = P1 ** N1         $     T20 = P2 ** N2
      R1 = (1.-P1)/P1       $     R2 = (1.-P2)/P2
      POFK3 = 0.
      A = II = N1           $     B = 1.
      IS = 0
      IF( K.GT.N2 ) 1,2
    1 IS = K - N2
    2 DO 30 I = IS,N1
      SM = 1.
      IF( II.GE.K ) 25,10
   10 SM = T2 = T20
      C = N2                $     D = 1.
      JS = K - II + 1
      DO 20 J = JS,N2
      T2 = T2 * (C/D) * R2
      C = C - 1.            $     D = D + 1.
   20 SM = SM + T2
   25 POFK3 = POFK3 + SM*T1
      T1 = T1 * (A/B) * R1
      A = A - 1.            $     B = B + 1.
   30 II = II - 1
      RETURN
      END
```